THE
ELEMENTARY PARTS
OF
Dr. SMITH's COMPLEAT SYSTEM OF OPTICKS,

SELECTED AND ARRANGED

FOR THE USE OF STUDENTS AT THE UNIVERSITIES:

TO WHICH ARE ADDED

IN THE FORM OF NOTES

SOME

EXPLANATORY PROPOSITIONS FROM OTHER AUTHORS.

R

CAMBRIDGE,

Printed by J. ARCHDEACON, Printer to the UNIVERSITY;

And ſold by T. & J. MERRILL, J. WOODYER, and J. NICHOLSON, at Cambridge;
J. & J. FLETCHER, and D. PRINCE, at Oxford; J. NOURSE, in the Strand,
and B. WHITE, in Fleetſtreet, London.

M. DCC. LXXVIII.

THE PREFACE.

WHEN the Editor of the following Treatiſe drew the outlines of his plan, he had no Intention either of printing ſo large a volume or of publiſhing any. His work was then calculated to facilitate the progreſs of ſcience in this ſociety alone, by accommodating the ſtudents with thoſe parts of Dr. SMITH's OPTICKS, which are explained to them in the lectures on that Branch of philoſophy. The ſcarcity of that Author's SYSTEM had long been a ſubject of complaint; and it was at length become impoſſible to procure a number of copies ſufficient for the wants of ſo large and induſtrious a ſociety. But the Editor had not proceeded far, before he was diverted from his firſt deſign, by being repeatedly informed that the ſame difficulty prevailed in almoſt every other College, and that no perſon could be found, who was willing to hazard a Republication of the whole of Dr. SMITH's Treatiſe. He therefore determined, though already oppreſſed by accumulated buſineſs, to extend his plan; rather than ſuffer the ſcarcity complained of to be an Impediment to Induſtry, or become an Excuſe for Idleneſs. To remove the former of theſe ill conſequences, and obviate the latter in the moſt effectual manner poſſible, he communicated his deſign to the Tutors of other Colleges, and added to the ſtock, he had formerly collected, all the parts of Dr. SMITH's Treatiſe, in which he found their lectures differed from his own.

But though the Editor was poſſeſſed of the requiſite materials, the chief difficulty ſtill remained. For the primitive form of theſe deranged Elements reſembled more the chaos of the poets, than a well-connected ſeries of philoſophical Reaſonings. His next attempt was therefore to reduce the whole into Syſtem, by claſſing its parts and

ranging the claſſes in ſuch order as ſhould beſt correſpond with the plan of Lectures given by the Tutors in this univerſity, whoſe exemplary diligence in the cultivation of ſcience entitles them to this and every other aſſiſtance. — After extricating theſe materials from diſorder and confuſion, in which his ſucceſs has far exceeded his expectations; he tried alſo to efface every other mark of deformity, and to give them the appearance of a new creation. But here he met with an inſuperable obſtacle. In ſome chapters of the Original the ſubject is popularly explained, and in others geometrically demonſtrated. It was therefore impoſſible to diffuſe over the Whole an uniformity of Style without endleſs Interpolations and a proportionable Delay, neither of which was conſiſtent with the Reaſons, which firſt induced him to undertake the Work.

Although the following Treatiſe was intended in its origin, and calculated in its progreſs, for the ſole uſe of the Academick, the Editor now ventures, on a review of its contents, to recommend it to the uſe of others, who deſire to be inſtructed in the firſt principles of Opticks. Theſe principles are demonſtrated in it with as much eaſineſs and perſpicuity, if not with ſo much elegance and accuracy, as in other works of the ſame nature; with ſo much eaſineſs, indeed, as to be intelligible to every Reader, who is previouſly furniſhed with the mere Rudiments of Geometry. And the order in which they are ranged, though peculiarly adapted to the original deſign for which they were ſelected, is in the Editor's opinion the moſt natural, in which the ſubject can be treated. The following is a ſlight ſketch of this arrangement. In the three firſt chapters are digeſted the articles which contain Dr. SMITH's Explanation of the general properties of *Light*, and Sir ISAAC NEWTON's Experiments to prove that Light conſiſts of different *Colours*. In the next chapter is traced the motion of a ſingle *Ray* of Light in its paſſage through refracting ſurfaces of a ſpherical figure, and in the four ſubſequent chapters the motion of a *Pencil* of Rays, till they unite again, after Reflection or Refraction, to form an *Image* of the object from whence they proceeded. And as we are made to ſee external objects by means of their Images formed upon the *Retina*, in the two next chapters are explained the whole ſtructure of the human *Eye*, the whole proceſs of *Viſion* with the naked Eye, what aſſiſtances the ſight receives from

Teleſcopes,

Teleſcopes, *Microſcopes*, and *Spectacles*, and the method of *conſtructing* theſe and other optical Inſtruments. The eleventh Chapter treats of the *Imperfection* in Teleſcopes and Microſcopes, which is cauſed by the *aberration* of Rays from their geometrical focus in conſequence of the ſpherical figure of a Lens and the different Refrangibility of different kinds of Light: and the twelfth chapter contains the known *Rules*, which, in conſtructing Teleſcopes, it is neceſſary to obſerve on account of theſe aberrations. The Elements of the ſcience being demonſtrated, they are applied in the two laſt chapters to ſolve the phænomena of *the Rainbow*, and *the annual aberration of the fixed ſtars*.

The only modern authors, who have treated geometrically the principles of this ſcience, and whoſe works are ſtill in print, are Mr. EMERSON and Mr. HARRIS. But the philoſophical writings of a ſuperior genius are ſeldom adapted to the capacity of an unaſſiſted Learner; and the demonſtrations of the former of theſe authors are not yet adopted by the Tutors in our univerſities. The elementary Treatiſe of the latter is entirely ſilent upon ſome of the moſt important ſubjects belonging to this ſcience, ſuch as the conſtruction of Teleſcopes and the phænomena of the Rainbow. Had Mr. Harris lived to finiſh his Work, it would have precluded the neceſſity of this and every other publication of the ſame extent.

It is not intended, in this character of the writings of others, to intimate in the ſlighteſt degree that the following Treatiſe is faultleſs. It contains many Inaccuracies and even ſome Errors, of which the Editor was fully ſenſible before he ſent it to the preſs, but was reſtrained from correcting them by the dread of Reprehenſion. The only method of correction was a compleat commentary on the Text, or frequent alterations of it. But, beſides that ſuch a commentary would have been as tedious and troubleſome as a new Treatiſe on the ſubject, there were other objections againſt it too obvious to be mentioned: and to have eraſed and corrected the Text of an eminent Writer however judiciouſly, might have been deemed by ſome an impertinent preſumption and an unjuſt Treatment of the author.

Theſe Inaccuracies might indeed have been prevented, and an uniformity introduced by compoſing a new ſyſtem of the ſame materials. But as the want of Dr. SMITH'S OPTICAL ELEMENTS was become too

too preſſing an Inconvenience to allow ſufficient time for executing a regular and well-digeſted plan, the Editor was reduced to the alternative either of garbling the works of that Author or of publiſhing ſome crudities of his own. Beſides, he had neither health nor leiſure to engage in any publication more laborious than the preſent: He prepared it for the preſs without the Trouble of copying any part of its contents, except a few propoſitions, which he has borrowed from other Books of eminence not eaſily to be procured, and which the Tutors in this univerſity have introduced into their lectures for a more ample explanation of what Dr. SMITH has but ſlightly touched upon. To have circumſcribed his collection of notes within a narrower limit would hardly have been poſſible; and he judged it more adviſeable that the Learner ſhould be left to conſult the entire works of *modern* authors, than that this volume ſhould be ſwelled with Extracts from their writings; it being of the greateſt aſſiſtance to the ſtudent, during his noviciate in philoſophy, to have the ſame Truth repreſented to him in a variety of lights. For the principal notes, which are ſubjoined to the following pages, the Reader is indebted to Dr. BARROW and DES CARTES.

Such is the Nature and Intention of the following treatiſe, and ſuch the Editor's apologies for preſuming to publiſh ſo irregular and incorrect a compoſition. But if theſe Reaſons be inſufficient to defend him againſt private cavils, he hopes the following conſiderations will ſecure him from public Cenſure: — that it could not be a deſire of Fame, which induced him to undertake the mechanical office of an *Editor*; or the Hope of Profit, to be the Inſtrument of a publication, the Expence of which muſt be great, and the purchaſers few; that he could have no View to his own Improvement in forming a ſyſtem of principles, which it has been his buſineſs for ſeveral years ſucceſſively to explain to others; and laſtly, had amuſement been his object, that he certainly would have directed his attention to ſome other province in the intellectual world leſs frequented by him than the preſent, leſs barren and more beautiful. His only motives were public utility and a deep ſenſe of the duty incumbent on every member of theſe Societies to promote the deſigns of thoſe venerable Benefactors, whoſe Endowments they have the honour and happineſs to participate.

SUBSCRIBERS.

Where no University is mentioned in the following List, that of Cambridge is understood.

A.

THE Right Honourable Lord Viscount Althorp of Trinity Coll.
Rev. John Arden, B. A. of St. John's Coll. and Rector of Oxnead, Norfolk
Pepper Arden, Esq; Fellow of Trinity Coll.
Rev. Mr. Atcherley, M. A. Head Master of Shrewsbury School
Rev. Mr. Atlay, M. A. Fellow of St. John's Coll.
Mr. Atwood, M. A. Fellow of Trinity Coll.

B.

Honourable Thomas Broderick, Member of Parliament for Midleton in Ireland
Peter Burrel, Esq; M.A. Member of Parliament for Haslemere, 3 Copies
Rev. James Browne, D.D. Master of Pembroke Hall
Mr. Babington of St. John's Coll.
Rev. Mr. Baines, M. A. Fellow of Sidney Coll.
Henry Banks, Esq; Fellow-Commoner of Trinity Hall
Rev. Mr. Baldwin, M. A. Fellow of St. Peter's Coll.
Mr. Bartholomew, B. A. of St. John's Coll.
Mr. Barnard, B. A. of Trinity Coll.
Mr. Barnard of St. John's Coll.
Mr. Barlow of Trinity Coll.
Rev. Edward Beadon, Chaplain to his Majesty
Rev. Richard Beadon, Public Orator in the University of Cambridge
Mr. Bell of St. John's Coll.
Mr. Beloe of Bene't Coll.
Mr. Bevil, B. A. of St. Peter's Coll.
Mr. Beeston of St. John's Coll.
Mr. Beauvoir of St. John's Coll.
Rev. Edward Blakeway, M. A. late Fellow of Magdalen Coll.
Mr. George Blount, B. A. of St. John's Coll.
Rev. Mr. Brand, M. A. Fellow of Christ's Coll.
Mr. Bramston of Trinity Coll.
Rev. Mr. Brundish, M. A. Fellow of Caius Coll.
Mr. Brereton of St. John's Coll.
Mr. Burne of St. John's Coll.
Mr. Burroughes of Queen's Coll.
Mr. Burroughes, B. A. of Caius Coll.
Mr. Butcher of St. John's Coll.
Rev. William Burslem, M. A. Fellow of St. John's Coll.

Right

C.

Right Honourable Earl of Clarendon

Frederick Cornewall, Esq; Diddlebury, 2 Copies

Frederick Cornewall, Esq; Member of Parliament for Leominster, 2 Copies

Rev. John Chevallier, D. D. Master of St. John's Coll.

Caius College Library, 2 Copies

Rev. John Carr, M. A. Fellow of St. John's Coll.

Mr. Carlyle of Christ's Coll.

Mr. Carpindale of St. John's Coll.

Mr. Capper of St. John's Coll.

Mr. Christian of St. John's Coll.

Mr. Clapham, B. A. of Clare Hall

Mr. John Clapham of Christ's Coll.

Rev. Mr. Clarke, M. A. Fellow of Clare Hall

Mr. Coffin of Sidney Coll.

Mr. Coffin of Clare Hall

Mr. William Colquitt of Christ's Coll.

Mr. Cookson, Fellow of St. John's Coll.

Mr. Cookson of St. John's Coll.

Mr. Constable, B. A. of St. Peter's Coll.

Rev. Mr. Coulthurst, B. A. of St. John's Coll.

Rev. F. H. Cornewall, Fellow of St. John's Coll. 2 Copies

Rev. Mr. Cook of St. John's Coll.

Rev. William Craven, B. D. President of St. John's Coll. and Professor of Arabic in the University of Cambridge

Rev. John Cranke, M. A. Fellow of Trinity Coll.

Rev. Thomas Craster, M. A. of St. John's Coll.

Mr. Curtis of St. John's Coll.

D.

Honourable Richard Dawson of St. John's Coll. 2 Copies

Somerset Davies, Esq; M. A. of St. John's Coll. 2 Copies

Mr. Charles Davy of Caius Coll.

Mr. Dealtry, B. A. of Trinity Coll.

Rev. Mr. Deane, B. D. late Fellow of St. John's Coll. 3 Copies

Rev. T. Drake, M. A. Fellow of St. John's Coll.

E.

Right Rev. Dr. Edmund Keene, Bishop of Ely

Rev. William Elliston, D. D. Master of Sidney Coll.

Rev. William Easton, M. A. Fellow of St. John's Coll. 3 Copies

Mr. Eaton of St. John's Coll.

Rev. Mr. Ellis, M. A. Fellow of Trinity Coll.

Thomas Eyton, Esq; of Wellington, 3 Copies

F.

Right Honourable Lord De Ferrars

Mr. E. Fagg of Clare Hall

Rev. Mr. Fallowfield, B. A. Fellow of Clare Hall

Rev.

Rev. Mr. Farnham, M. A. of St. John's Coll.
Rev. Charles Favell, M. A. Fellow of Clare Hall
Rev. James Fawcett, M. A. Fellow of St. John's Coll.
Rev. Thomas Ferris, B.D. Fellow of St. John's Coll.
Mr. Ferraby of Clare Hall
Rev. John Fiſher, M. A. Fellow of St. John's Coll.
Rev. Mr. Fiſher, M. A. Fellow of Caius Coll.
Rev. Robert Fiſke, M. A. Fellow of St. John's Coll.
Mr. Forby of Caius Coll.
Mr. Frend of Chriſt's Coll.
Mr. Froſt of Caius Coll.
A. Frere, Eſq; M. A. of St. John's Coll.
Rev. Edward Frewen, M. A. Fellow of St. John's Coll.

G.

Thomas Giſborne, Eſq; Fellow-Commoner of St. John's Coll.
Rev. Dr. Gordon, Precentor of Lincoln.
James Gordon, Eſq; Fellow-Commoner of St. John's Coll. 2 Copies
Rev. Mr. Gould, M. A. Fellow of Clare Hall
Mr. Greenwood, B. A. of St. John's Coll.
Rev. Mr. Gretton, B. A. of Trinity Coll.
Mr. Griffith of St. John's Coll.
Mr. Grigſon of Caius Coll.
Rev. Harry Grove, M. A. Fellow of St. John's Coll. and Junior Proctor in the Univerſity of Cambridge
Robert Lovell Gwatkin, Eſq; B. A. Fellow-Commoner of St. John's Coll. 2 Copies

H.

Right Honourable Lord Hyde
Noel Hill, Eſq; Member of Parliament for the County of Salop, 4 Copies
Rev. Anthony Hamilton, D. D. Archdeacon of Colcheſter
Mr. Hale of Clare Hall
Rev. Mr. Hall, B. A. of St. John's Coll.
Mr. Hammond, B. A. of Queen's Coll.
Mr. Hammond of St. John's Coll.
Mr. William Hamond of St. John's Coll.
Mr. Hanley of Chriſt's Coll.
Mr. Harley of Trinity Coll.
Mr. George Harriſon of St. John's Coll.
Mr. Hart, B.A. of St. John's Coll.
Mr. Heath of St. John's Coll.
Mr. Heberden, Fellow of St. John's Coll. 3 Copies
Mr. Henſon of St. John's Coll.
Mr. Heſlop, B. A. of Sidney Coll.
Rev. Mr. Hetley, M. A. of St. John's Coll.
Rev. John Hey, B. D. Fellow of Sidney Coll.
Rev. S. Hey, M. A. Fellow of Magdalen Coll.
Mr. Holcombe of St. John's Coll.
Rev. B. Holmes, M. A. Fellow of St. John's Coll.
Mr. How of St. Peter's Coll.

Rev. T. Hughes, B. A. of St. John's Coll.
Rev. Lewis Hughes, M. A. Fellow of St. John's Coll.
Rev. Mr. Humphries, M. A. Second Maſter of Shrewſbury School
Rowland Hunt, Eſq; of Boreaton
Rev. Mr. Hunter, B. D. Fellow of Sidney Coll.

J.

Mr. James, M. A. Fellow of King's Coll.
St. John's College Library
Rev. Samuel Johnſon, Third Maſter of Shrewſbury School
Mr. Jones of Trinity Coll.
Rev. Mr. Jordan, M. A. Fellow of Queen's Coll.
Mr. Jordan of Pembroke Hall
Rev. Mr. Jowett, LL. B. Fellow of Trinity Hall

K.

Mr. Kerrick, M. A. Fellow of Magdalen Coll.
Mr. Kilvert, B. A. of Emanuel Coll.
T. Kinnerſley, Eſq; of Leighton, 2 Copies
Mr. R. Kynaſton of St. John's Coll.
Mr. Ed. Kynaſton of St. John's Coll.

L.

Mr. S. Lawrence, M. A. Fellow of St. John's Coll. 2 Copies
Mr. Lawrence of Chriſt's Coll.
Mr. Latter of St. John's Coll.
Mr. S. Leacroft, London, 3 Copies
Charleton Leighton, Eſq; of Loton, 3 Copies
Thomas Lempriere, Eſq; Jerſey
Mr. Lens of St. John's Coll.
Mr. Littlehales of St. John's Coll.
Mr. Lodington, B. A. of Sidney Coll.
Mr. Lovering of St. John's Coll.
Mr. Loxham of St. John's Coll.
Mr. Lund of St. John's Coll. 2 Copies

M.

Right Honourable Lord Viſcount Midleton
Right Honourable Lord Maxwell
Rev. Owen Manning, B. D. of Godalmin
Mr. Mantell, B.A. of Bene't Coll.
Rev. Mr. Marſh, M. A. Fellow of Trinity Coll.
Mr. Marſh of St. John's Coll.
Rev. Mr. Metcalfe, M. A. Fellow of Chriſt's Coll.
Rev. Iſaac Milner, M. A. Fellow of Queen's Coll.
George Milner, Eſq; Fellow-Commoner of St. John's Coll.
Mr. Moiſes of Trinity Coll.
Rev. Mr. Mountain, M. A. of Caius Coll.
Rev. Mr. Mounſey, M. A. Fellow of Jeſus Coll. and Moderator in the Univerſity of Cambridge
Mr Munnings of Caius Coll.

N.

Rev. Charles Newling, Prebendary of Litchfield and Cvo.
Mr. William Nelſon, B. A. of Chriſt's Coll.

Mr. Nesfield of Caius Coll.
Mr. North, M.A. Fellow of Caius Coll.

O.

Rev. Mr. Owen, B. A. of Trinity Coll.

P.

Right Honourable Earl of Powis, 4 Copies
Mr. Paley of Chriſt's Coll.
Mr. Palmer, B. A. of St. John's Coll.
Rev. Mr. Parkinſon, M. A. Fellow of Chriſt's Coll.
Rev. William Pearce, M. A. Fellow of St. John's Coll.
J. Pennington, M. D. Fellow of St. John's Coll.
Mr. Pyndar, Fellow of Queen's Coll.
Mr. Popple, B. A. of Trinity Coll.
Rev. Mr. Porter, M. A. Fellow of Trinity Coll.
Mr. William Pott of St. John's Coll.
Charles Poole, Eſq; Fellow-Commoner of Pembroke Hall
Rev. Mr. Poſtlethwaite, M. A. Fellow of Trinity Coll.
Thomas Jelf Powys, Eſq; of Berwick, 2 Copies
Rev. Edward Powys, Fellow of St. John's Coll. 2 Copies
Mr. Praed of St. John's Coll.
Rev. Mr. Prettyman, M. A. Fellow of Pembroke Hall
Rev. R. Pritchett, B. D. Fellow of St. John's Coll.
Mr. Prichard of St. John's Coll.
Mr. Pym of Chriſt's Coll.

Q.

Queen's College Library

R.

Rev. Mr. Raines, M. A. Fellow of Jeſus Coll.
Mr. Raper of St. John's Coll.
Mr. Reaſton of St. John's Coll.
Mr. Rippon of Trinity Coll.
Mr. J. Rocke of St. John's Coll. 2 Copies
M. Robinſon of St. John's Coll.
Rev. John Robinſon, M. A. late Fellow of Clare Hall

S.

Right Honourable Lord St. John of St. John's Coll.
Honourable St. Andrew St. John of St. John's Coll.
Rev. Mr. Sandiford, M. A. Fellow of Trinity Hall
Rev. Mr. Seale, M. A. Fellow of Chriſt's Coll.
Mr. Sheild of St. John's Coll.
Mr. Shipley, B. A. of Clare Hall
Mr. Skeels, B. A. of Pembroke Hall
Rev. William Smith, M. A. Fellow of St. John's Coll.
Mr. Joſeph Smith of Caius Coll.
Mr. Sneyd, B. A. of St. John's Coll.
Mr. Sparhauke, B. A. of St. John's Coll.
Rev. Mr. Squire, M. A. Fellow of Caius Coll.
Mr. Standen of Queen's Coll.
Mr. Sturges of Clare Hall
Rev. Mr. Stephenſon, M. A. Fellow of Clare Hall, 2 Copies
Rev. Thomas Starkie, M.A. Fellow of St. John's Coll.
Mr. Sutton of St. John's Coll.

T.

Honourable John Townſhend, M. A. of St. John's Coll.

William Tayleur, Eſq; of Shrewſbury

Rev. Mr. Therond, M. A. Fellow of Trinity Coll.

Mr. Tremenheere of Pembroke Hall

Mr. Tucker of Trinity Coll.

Robert Tunſtall, Eſq; Fellow-Commoner of Trinity Coll.

Rev. Joſeph Turner, M. A. Fellow of Pembroke Hall

George Turner, Eſq; Fellow-Commoner of Trinity Coll.

Mr. Twigge, B. A. of St. John's Coll.

Richard Tylden, Eſq; B. A. Fellow-Commoner of Jeſus Coll.

V.

Honourable John Charles Villiers, M. A. of St. John's Coll.

Mr. Vince, B. A. of Sidney Coll.

W.

Rev. William Wade, Fellow of St. John's Coll.

Mr. Wakefield, Fellow of Jeſus Coll.

Rev. Mr. Walſby, M. A. Fellow of Bene't Coll.

Rev. Peploe Ward, M. A. Fellow of Queen's Coll. and Prebendary of Ely

Rev. Richard Watſon, D.D. F.R.S. Regius Profeſſor of Divinity in the Univerſity of Cambridge, and Prebendary of Ely

Thomas Lucas Wheeler, Eſq; Fellow-Commoner of Trinity Coll.

Mr. Whinfied, B. A. of St. Peter's Coll.

Rev. George Whitmore, M. A. Fellow of St. John's Coll.

Mr. George Wilſon, B. A. of Chriſt's Coll.

Rev. John Wingfield, B.A. of St. John's Coll.

Rev. Mr. Woodrooffe, M. A. Fellow of Clare Hall

Mr. Woodman of Trinity Coll.

Rev. James Wood, M.A. Fellow of St. John's Coll.

Mr. Wright, Fellow of St. John's Coll.

Z.

Rev. Thomas Zouch, M. A. Rector of Wycliffe

THE

ELEMENTARY PARTS

OF

Dr. SMITH's COMPLEAT SYSTEM OF OPTICKS.

THE

C O N T E N T S.

ERRATA.

E R R A T A.

Article
40. line 6. *for* reflected, *read* refracted.
47. l. 8. *for* the refracting, *r.* their refracting.
64. in the margin *for* Fig. 56 to 59. *r.* Fig. 56 & 59.
70. at the beginning of the 2d part, the word DEFINITIONS is wanting.
79. line 7. *for* the Focus Q. *r.* the Focus q.
129. l. 1. *for* E q, *r.* E Q.
140. l. 34. *for* perhaps a hundred miles or more, *r.* some hundreds of miles.
172. l. 8. *for* p l, *r.* q l.
181. l. 21. *for* T c s, *r.* T c S.
228. l. 1. *dele* and double microscopes.
244. in the 4th column of the Table, *for* 199 and 398, *r.* 200 and 400.
258. in the margin, *for* Art. 31. *r.* Newton's Opt. p. 114. 8vo.

Plate
5. Fig. 67. C is wanting at the vertex of the parabola.
9. Fig. 127. r is wanting at the upper extremity of p q r.
11. Fig. 157. C is wanting where E O cuts the circle.

THE

OPTICKS.

CHAP. I.

CONCERNING LIGHT.

1. WHOEVER has considered what a number of properties and effects of light are exactly similar to the properties and effects of bodies of a sensible bulk, will find it difficult to conceive that light is any thing else but very small and distinct particles of matter *: which being incessantly thrown out from shining substances, and every way dispersed by reflection from all others, do impress upon our organs of seeing that peculiar motion, which is requisite to excite in our minds the sensation of light. But for the present purpose it is sufficient to observe that light consists of parts, both successive in the same lines and contemporary in several lines: because in the same place, you may stop that which comes one moment, and let pass that which comes presently after; and at the same time, you may stop it in one place, and let it pass in another. For that part of the light which is stopt cannot be the same with that which is let pass. Light consists of parts.

2. The least light or part of light, which may be stopt alone without the rest of the light, or propagated alone, or do or suffer any thing alone, which the rest of the light doth not or suffereth not, is called a Ray of light †. That rays of light are straight, is evident enough from the shadows of bodies; or from the appearance of light passing through little holes into a dark room full of dust or smoke; or because bodies cannot be seen through the bore of a bended A ray of light what and how considered.

* Newt. Opt. Qu. 29. p. 345. 8°. Edit.

† Newton's definition. Opt. p. 1.

pipe; or becaufe they ceafe to be feen by the interpofition of other bodies, as the fixt ftars by the interpofition of the moon and planets; and the parts of the fun by the interpofition of the Moon, Mercury or Venus. Rays of light may therefore be reprefented by ftraight lines, not Mathematical but Phyfical, which are defcribed by the motion of the parts or particles of light: and the point which a ray poffeffes in falling upon any furface may be confidered as a Phyfical Point.

The manner of reflection and refraction of a ray defcribed. Fig. 1.

3. When a ray of light falls obliquely upon a fmooth polifhed furface, it is turned out of its way either by reflection or refraction in the following manner. Imagine the paper upon which this figure is drawn to be perpendicular to the furface of ftagnating water, and to cut it in the line *RS*, and that a ray of light, coming in the air along the line *AC*, falls upon *RS* at the point *C*. Then fuppofing the line *PCQ* to be perpendicular to the furface of the water, if the ray be reflected, or turned back at *C* into the air again, it will defcribe a ftraight line *CB*, inclined to the perpendicular *CP* at an angle *PCB* exactly equal to the angle *PCA*.

Fig. 2.

But if the ray that came along *AC* goes into the water at *C*, it will not proceed ftraight forward, but being refracted or bent at *C*, it will defcribe another ftraight line *CE* inclined to the perpendicular *CQ* at a leffer angle *ECQ* than the angle *ACP*; and the line *CE* will always be fo fituated, that when any circle, defcribed about the center *C*, cuts the line *CA* in *A* and *CE* in *E*, the perpendiculars *AD* and *EF*, drawn from *A* and *E* to the line *PQ*, fhall always bear the fame proportion to each other; whatever be the magnitude of the angle *ACP*. In water the line *EF* is always three quarters of *AD*.

Angles and fines of incidence and refraction what. Fig. 1, 2.

4. In both thefe cafes the line *AC* is called the Incident Ray, *CB* the Reflected Ray, *CE* the Refracted Ray, *C* the point of incidence, *PCQ* the perpendicular (at the point) of incidence, the angle *ACP* the Angle of Incidence, *BCP* the Angle of Reflection, *ECQ* the Angle of Refraction; the line *AD* the Sine of Incidence, that is, of the angle of incidence; and *EF* the Sine of Refraction, that is, of the angle of refraction.

A medium what.

5. Empty fpace, or any tranfparent body, is called a Medium; and mediums are denfer in proportion as they are heavier bulk for bulk; and their power to reflect and refract light is found to be greater in proportion as they are denfer, very nearly *.

Laws of reflection and refraction what.

6. The foregoing properties of Reflection and Refraction being difcovered and eftablifhed by repeated experiments upon light and bodies of all forts both fluid and folid, without any exception yet

* Newt. Opt. p. 245. 8°.

known;

known; and being the principal foundation of the whole ſcience of Opticks, are called the Laws of Reflection and Refraction; and are expreſſed by Sir *Iſaac Newton* in the following words.

7. *The angles of reflection and refraction lye in one and the ſame plane with the angle of incidence*; that is, in the plane drawn through the incident ray and the perpendicular at the point of incidence, as repreſented in the figures, 1, 2. First law.

8. *The angle of reflection is equal to the angle of incidence.* Second law.

9. Hence it follows that the incident and reflected rays are equally inclined to the reflecting plane; that is, the angles *ACR* and *BCS* are equal; as appears by taking the equal angles *PCA* and *PCB* from the equal angles *PCR* and *PCS*. First conſequence. Fig. 1.

10. It follows alſo that when the incident ray is perpendicular to the reflecting ſurface, it ſhall be reflected directly back along the ſame perpendicular; as appears by diminiſhing the equal angles of incidence and reflection till the rays *AC*, *CB* coincide with the perpendicular *CP*. Second conſequence.

11. *If the reflected or refracted ray be returned directly back to the point of incidence, it ſhall be reflected or refracted into the ſame line before deſcribed by the incident ray.* Third law.

12. *Refraction out of a rarer medium into a denſer* [a] *is made towards the perpendicular*; *that is, ſo that the angle of refraction be leſs than the angle of incidence.* Fourth law. [a] Art. 5.

13. *The ſine of incidence, AD, is to the ſine of refraction, EF, either accurately or very nearly in a given ratio*; that is, ſuppoſing any other incident ray *aC* to be refracted into the line *Ce*, and the ſines *ad* and *ef* to be drawn perpendicular to *PQ*, the ratio of *ad* to *ef* is the ſame as the ratio of *AD* to *EF*. It is found by experience, that if the refraction be made out of air into water, the ſine of incidence of red light is to the ſine of its refraction as 4 to 3: if out of air into glaſs as 17 to 11, or nearly as 3 to 2. In light of other colours the ſines have other proportions, but the difference is ſo little that it ſeldom need be conſidered. Fifth law. Fig. 4.

14. Hence it appears by inſpection of the figures (2, 3, 4.) that when the angle of incidence *ACP* is increaſed, the correſponding angle of refraction *ECQ* will alſo be increaſed; becauſe the ratio of their ſines, *AD*, *EF*, cannot continue the ſame unleſs they be both increaſed. Conſequently if two angles of incidence be equal to each other, the angles of refraction will alſo be equal to each other. On the contrary, when the angle of incidence is diminiſhed, the angle of refraction will alſo be diminiſhed; inſomuch that when one of theſe angles becomes infinitely ſmall the other alſo becomes infinitely ſmall. First conſequence.

Second conſequence.

15. And ſo it comes to paſs that when the incident ray coincides with the perpendicular to the refracting ſurface, it will proceed ſtraight forward into the other medium without any bending at all.

Third conſequence.

16. From which it is reaſonable enough to conclude back again, that while the angle of incidence is continually increaſing, the refracted ray will be continually more and more bent and diverted from the courſe of the incident ray produced: I mean if *AC* be continued to *G*, the arc *EG* and the angle *ECG* will continually increaſe *: eſpecially conſidering that when the angle of incidence in air becomes very nearly a right one, and conſequently the incident ray goes almoſt parallel to the ſurface of the water, this ray is as much bent at *C* into the line *CE* as the 3d figure repreſents. In which *EF*, the ſine of refraction, being always three quarters [a] of *AD*, is now three quarters of the radius of the circle. Hence we find † that this angle of refraction, *ECQ*, is about $48\frac{1}{2}$ degrees: and ſo the angle *ECS* (being its complement to 90 degrees) is about $41\frac{1}{2}$ degrees; which in this caſe meaſures the deviation of the ray from its firſt courſe along the ſurface of the water. The deviation at the ſurface of glaſs is greater than at the ſurface of water; the ratio of the ſines

Fig. 2. Fig. 3. [a] Art. 13.

* PROPOSITION. *The greater the angle of incidence is, the greater will be the angle of deviation.*

1. In the caſe of reflection, the angle of deviation is that angle which is contained by the incident and reflected rays. But if the angle of incidence *ACP* increaſes, the double of that angle, or *ACB* [a], muſt increaſe alſo.

Fig. 1. [a] Art. 8.

2. In the caſe of refraction, the angle of deviation is that angle which is contained by the refracted ray and the courſe of the incident ray produced. Draw *QBP* perpendicular to the refracting plane *EBF*; let *ABG*, *DBH*, be two incident rays, of which *AB* falls more obliquely than *DB*; and let *Ba*, *Bd*, be the directions in which they move reſpectively after refraction: I affirm that the angle *GBa* is greater than *HBd*.

Fig. 5.

In the perpendicular *QB* produced take any point *P*, and upon the diameter *BP* deſcribe the ſemicircle *BGP*, cutting *AB*, *DB* produced in the points *G*, *H*, and *Ba*, *Bd* in the points *a*, *d*: Join *PG*, *Pa*, *PH*, *Pd*. And *PGB*, *PaB*, *PHB*, *PdB* being right angles [b], *PG*, *PH* are the right ſines of the angles of incidence *PBG*, *PBH*, to the radius *BP*; alſo *Pa*, *Pd* are the right ſines of the angles of refraction *PBa*, *PBd*, to the ſame radius. Next, make the angle *GBK* equal to *HBd*, or the arc *GK* equal to the arc *Hd*; and join *Gd*, cutting *PK* in *X*: laſtly, draw the chords *Ga*, *Hd*. Now the angles *PGd*, *PHd* being equal [c], and the angles *GPK*, *HPd* being alſo equal, by conſtruction, the triangles *GPX*, *HPd* are ſimilar. Therefore *PG* is to *PX* as *PH* to *Pd*: but, by the law of refraction [d], *PH* is to *Pd* as *PG* to *Pa*; conſequently *PG* is to *PX* as *PG* to *Pa*, and therefore *PX* equals *Pa*. But *PX* is leſs than *PK*, becauſe the chord *Gd* lies wholly within the circle; and therefore *Pa* is leſs than *PK*: conſequently *PK* cuts the angle *GPa*, and the arc *Ga* is greater than the arc *GK*, or *Hd*. Wherefore the angle *GBa* is greater than *HBd*. Q. E. D.

[b] Euc. III. 31. [c] Euc. III. 27. [d] Art. 13.

In the preceding demonſtration, the angle of incidence is ſuppoſed to be greater than the angle of refraction: But the truth of the propoſition may be eaſily deduced from it, when the ray paſſes out of a denſer medium into a rarer. For if *aB*, *dB* are made the incident rays, *BA*, *BD* become the refracted rays [d]; and therefore *aBG*, *dBH* are ſtill the angles of deviation.

† By a Table of ſines.

being

being greater, that is, as 3 to 2, or nearer as 31 to 20. Hence we find that the angle *ECQ* is about 40 and *ECS* about 50 degrees.

17. The bending and deviation is the same when the ray goes back again along the same lines *EC*, *CA*; and if an angle of incidence *eCQ* be any thing greater than about $48\frac{1}{2}$ degrees in water, or any thing greater than about 40 in glass, this ray *eC* will not be refracted into air, but will be reflected into the line *Cf*, making the angle of reflection *QCf* equal to the angle of incidence *QCe*. Refraction changed into reflection.

18. The truth of these laws and of all the consequences drawn from them may be easily examined in the manner following. Upon a smooth board *KLMN*, about a center *C* with any opening of the compasses (the larger the better) describe a circle *PRQS*; and having drawn two diameters *PQ* and *RS* perpendicular to each other, from the point *P*, with any opening of the compasses, cut off equal arches *PA*, *PB*, and draw the lines *CA*, *CB*; then sticking three pins perpendicular to the board at the points *A*, *B*, *C*, dip the board into water as far as the line *RS*; and holding it perpendicular to the surface of the water, look along the pins *A*, *C*; and an image of the pin *B* will appear in the water in the line *AC* produced. Which shews that the ray which came from the pin *B* is reflected from the water, at the point *C*, along the line *CA* to the eye of the spectator. If the pin at *C* touches the water, it will disturb the smoothness of its surface; and therefore it is better not to place it in the center, but a little higher in the line *CA*. The event will be the same if the reflection be made by any other fluid or solid body, as may be tried by cutting off the lower semicircle, and by placing the diameter, *RS*, of the upper semicircle upon the surface of the solid. Experimental proof of these Laws of reflection and refraction. Fig. 4.

Upon the same board draw the line *AB* cutting *CP* in *D*, and from the lines *DB* and *CS* cut off *DH* and *CI*, each equal to three quarters of *DA*, and through the points *H*, *I*, draw the line *HIE*, cutting the circumference in *E*; and the perpendicular *EF* drawn from *E* upon *PQ* will be equal to *DH*, or three quarters of *DA*. Then stick another pin at *E*, and the board being dipped into water, as before, the pin at *E* will appear to the eye to be in the same line with the pins at *A* and *C*. Which shews that the ray which comes from the pin *E* is so refracted at *C*, as to advance to the eye along the line *CA*; and therefore when the refraction is made out of water into air, *EF* the sine of incidence, is to *AD* the sine of refraction, as 3 to 4. If other pins be fixed any where in the line *CE*, they will all appear in the line *AC* produced: and the whole line *CE* will appear in the water as if it were a continuation of *AC* straight forward. Which shews that the ray which comes from the pin *E*, describes a straight line in the water; and that it is bent at the surface only. On the con-

contrary, if an opportunity be taken when the Sun is juſt ſo high, that the ſhadow of the pin *A* ſhall coincide with the line *AC*, the refracted ſhadow will coincide with the line *CE*. Or whatever be the Sun's height, move the pin *A* higher or lower till the ſhadow falls upon the center *C*, and there fix it, ſuppoſe at *a*; then ſticking the compaſſes into any point of the refracted ſhadow, take up the board, and through this point and the center *C* draw a line *Ce*, cutting the circle in a new point *e*; and the ratio of the new perpendiculars, *ad* and *ef*, will be the ſame as before; that is, as 4 to 3, as near as can be meaſured.

This proof applied to ſpherical ſurfaces. Fig. 6, 7.

19. Laſtly it is to be obſerved, that a ray of light is reflected or refracted at a ſpherical ſurface according to the ſame laws as if it were reflected or refracted at a plane, touching the ſpherical ſurface at the point of incidence. Let *AC* be a ray of light falling upon any point *C* of a ſpherical ſurface *MCN*, repreſented by the arc *MCN*, whoſe center is *O*; through the points *O* and *C* draw the line *PQ*, and the line *RCS* perpendicular to it, repreſenting a plane ſurface touching the ſpherical ſurface at *C*. Now becauſe a ray of light is conſidered as a phyſical line, and is refracted or reflected at a phyſical point[a], which is common to both ſurfaces, *MCN* and *RCS*, it follows that the refracted or reflected ray will take the ſame courſe in both caſes. And this argument is alſo confirmed by univerſal experience.

[a] Art. 18.

An object what and how it radiates.

20. As rays of light are inceſſantly thrown out and diſperſed in all poſſible directions from every point of a luminous body; ſo when they illuminate other bodies, on which they fall, they are alſo inceſſantly thrown back from every point of theſe bodies. For the points of opake bodies ſo enlightened are viſible to the eye at any point of ſpace and in any point of time, as well as the points of the luminous body that enlightened them. The numberleſs rays which flow from all viſible bodies, called objects, may be methodically diſtributed in this manner. The ſurface of the object is conſidered as conſiſting of phyſical lines, and theſe lines as conſiſting of phyſical points, and theſe points are conceived to radiate all manner of ways. It is uſual to make uſe of nothing elſe for an object but a phyſical line. For by how much that line is increaſed or diminiſhed in apparent magnitude or brightneſs or diſtinctneſs, ſo much the diameter or length of any object, in its place, would be increaſed or diminiſhed.

A focus, pencil, parallel rays what. Fig. 10.

21. The point *Q* from which rays diverge, or towards which they converge (being made to go back towards the ſame point though they may never meet at it) is called their focus. And in both caſes any parcel of theſe rays, as *QBC*, or *QBA*, conſidered apart from the reſt, is called a pencil of rays; and theſe rays are ſaid to belong to that

that focus, whether it be near at hand or at an immenſe diſtance; and in the latter caſe the rays are called, and conſidered as, parallel or equidiſtant from each other; becauſe the difference of their diſtances at any two given places is inſenſible.

22. The 8th and 9th figures repreſent a pencil of rays, *QC*, which falling in parallel lines upon a plane poliſhed ſurface, repreſented by the line *ACB*, are reflected from it into as many other parallel lines, *Cq*; becauſe they are inclined to that plane juſt as much as the incident rays were inclined to it[a].

Reflection of a pencil of parallel rays at a plane ſurface.

[a] Art. 9.

23. In the 11th figure *QC* repreſents a pencil of parallel rays falling obliquely upon a ſtraight line *ACB*, or upon a plane ſurface repreſented by it, which after refraction are alſo parallel among themſelves; every one being equally bent. Becauſe when the angles of incidence are all equal among themſelves the angles of refraction are alſo equal among themſelves[b]. For the ſame reaſon if theſe rays be refracted again at another plane, either parallel or oblique to the former, they will ſtill be parallel among themſelves after every refraction. In ſtrictneſs this is only to be underſtood of rays of the ſame colour: as will be explained in the next chapter.

Refraction of a pencil of parallel rays at a plane ſurface.

[b] Art. 14.

Fig. 11, 12.

24. Let the light which flows from a point *A* and paſſes through a ſquare hole *bcde* be received upon a plane, *BCDE*, parallel to the plane of the hole; or if you pleaſe let the figure *BD* be the ſhadow of the plane *bd*; and when the diſtance *AB* is double of *Ab*, the length and breadth of the ſhadow *BD* will each be double the length and breadth of the plane *bd*; and treble, when *AB* is treble of *Ab*; and ſo on: which may be eaſily examined by the light of a candle placed at *A*.

Experiment. To ſhew that the breadths of a pencil of diverging rays are as their diſtances from the focus.

Fig. 13.

25. Therefore the ſurface of the ſhadow *BD*, at the diſtance *AB* double of *Ab*, is diviſible into four ſquares, and at a treble diſtance, into nine ſquares, ſeverally equal to the ſquare *bd*, as repreſented in the figure. The light then which falls upon the plane *bd*, being ſuffered to paſs to a double diſtance, will be uniformly ſpread over four times the ſpace, and conſequently will be four times thinner in every part of that ſpace, and at a treble diſtance it will be nine times thinner, and at a quadruple diſtance ſixteen times thinner, than it was at firſt; and ſo on according to the increaſe of the ſquare ſurfaces *bcde*, *BCDE*, &c, or of the ſquare ſurfaces *Abfg*, *ABFG*, &c, built upon the diſtances *Ab*, *AB*, &c. Conſequently the quantities of this rarified light received upon a ſurface of any given ſize and ſhape whatever, removed ſucceſſively to thoſe ſeveral diſtances, will be but one quarter, one ninth, one ſixteenth, of the whole quantity received by it at the firſt diſtance *Ab*. Or in general words the denſities and quantities

Hence the denſity and quantity of light received upon a given plane are reciprocally as the ſquares of its diſtances from the luminous body.

quantities of light, received upon any given plane, are diminiſhed in the ſame proportion as the ſquares of the diſtances of that plane, from the luminous body, are increaſed: and on the contrary, are increaſed in the ſame proportion as thoſe ſquares are diminiſhed. For the lights of the ſeveral points of the body, which ſeverally follow this rule, will compoſe a light which will ſtill follow the ſame rule.

CHAP. II.

CONCERNING THE ORIGIN AND CAUSE OF COLOURS.

Deſign.

THE preceding chapter contains ſuch properties as belong to all kinds of light; in this are related ſome of the experiments by which Sir *Iſaac Newton* diſcovered that light conſiſts of different colours.

A priſm what. Fig. 14, 15.

26. A glaſs priſm is a body, ſhaped like a wedge, that has three edges, being bounded with two equal and parallel triangular ends *ABC* and *abc*, and three plane and well poliſhed ſides, which meet in three parallel lines *Aa*, *Bb*, *Cc*, running from the three angles of one end to the three angles of the other: and when it is viewed endways it is repreſented only by a triangle *ABC*, as in the 15th figure.

I. Experiment. A deſcription of the ſun's image made by a priſm. Newt. Opt. p. 21. Fig. 15.

27. In a very dark chamber at a round hole *F*, about one third of an inch broad, made in the ſhut of a window, I placed a glaſs priſm *ABC* whereby the beam of the ſun's light *SF*, which came in at that hole, might be refracted upwards, toward the oppoſite wall of the chamber, and there form a coloured image of the ſun, repreſented at *PT*. The axis of the priſm, (that is the line paſſing through the middle of the priſm, from one end of it to the other end, parallel to the edge of the refracting angle) was in this and the following experiments perpendicular to the incident rays. About this axis I turned the priſm ſlowly, and ſaw the refracted light on the wall, or coloured image of the ſun, firſt to deſcend, and then to aſcend. Between the deſcent and aſcent when the image ſeemed ſtationary, I ſtopped the priſm and fixt it in that poſture.

Then I let the refracted light fall perpendicularly upon a ſheet of white paper *MN*, placed at the oppoſite wall of the chamber, and obſerved the figure and dimenſions of the ſolar image, *PT*, formed on the paper by that light. This image was oblong and not oval, but terminated by two rectilinear and parallel ſides and two ſemicircular ends. On its ſides it was bounded pretty diſtinctly, but on its

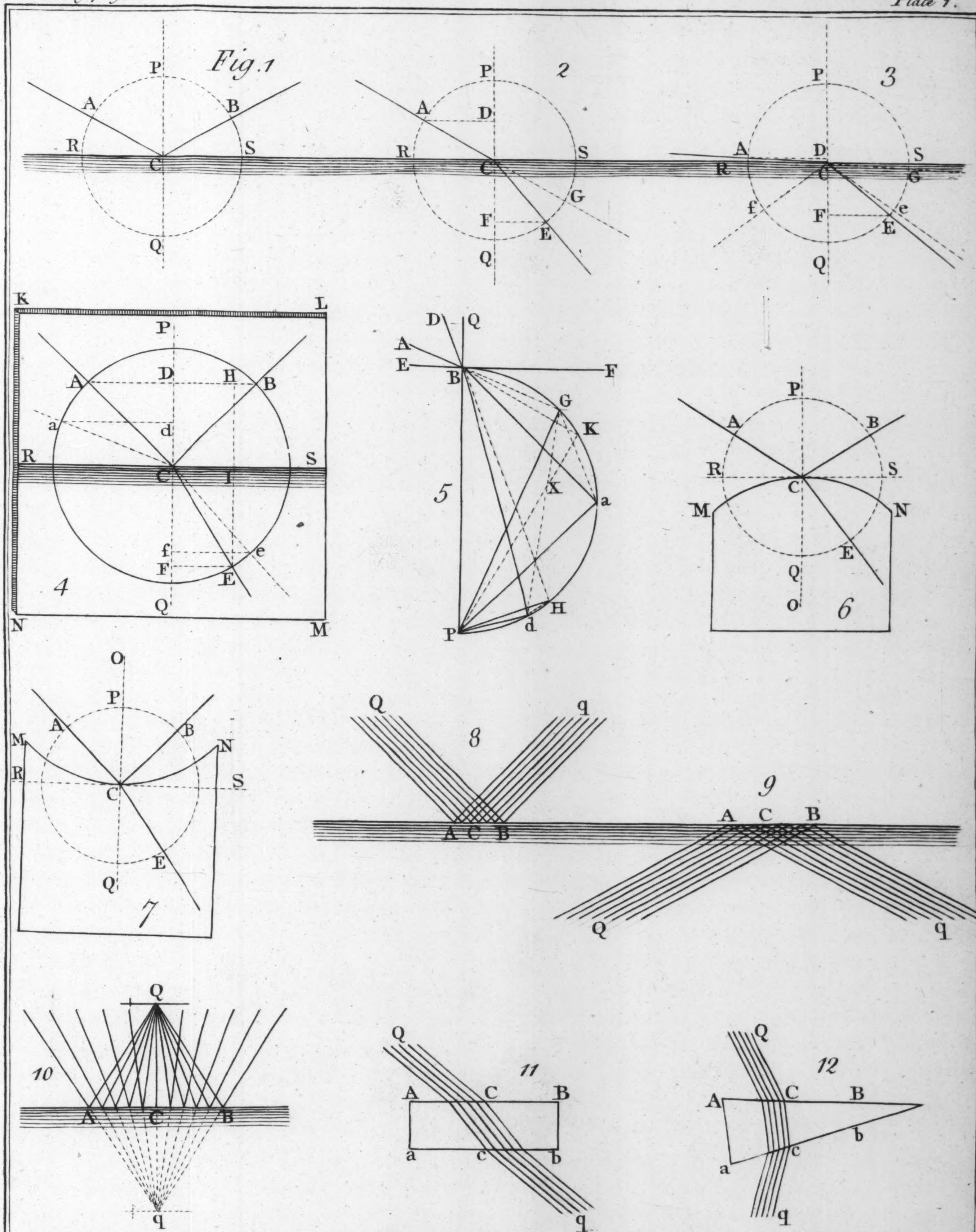


Fig. 1
2
3
4
5
6
7
8
9
10
11
12

its ends very confusedly and indistinctly, the light there decaying and vanishing by degrees. At the distance of 18½ feet from the prism the breadth of the image was about 2⅛ inches, but its length was about 10¼ inches, and the length of its rectilinear sides about 8 inches; and *ACB* the refracting angle of the prism, whereby so great a length was made, was 64 degrees. With a less angle the length of the image was less, the breadth remaining the same. It is farther to be observed that the rays went on in straight lines from the prism to the image, and therefore at their going out of the prism had all that inclination to one another from which the length of the image proceeded. This image *PT* was coloured, and the more eminent colours lay in this order from the bottom at *T* to the top at *P*; red, orange, yellow, green, blue, indigo, violet; together with all their intermediate degrees in a continual succession perpetually varying.

28. Our author concludes from this experiment, and some others to be mentioned hereafter, that the light of the sun consists of a mixture of several sorts of coloured rays, some of which at equal incidences are more refracted than others, and therefore are called more refrangible. The red at *T*, being nearest to the place *Y*, where the rays of the sun would go directly if the prism was taken away, is the least refracted of all the rays; and the orange, yellow, green, blue, indigo and violet are continually more and more refracted, as they are more and more diverted from the course of the direct light. For by mathematical reasoning he has proved, that when the prism is fixt in the posture above mentioned, so that the place of the image shall be the lowest possible, or at the limit between its descent and ascent, the figure of the image ought then to be round like the spot at *Y*, if all the rays that tended to it were equally refracted. Therefore seeing by experience it is found that this image is not round, but about 5 times longer than broad, it follows that all the rays are not equally refracted.

Hence the sun's rays are differently refrangible.

For the discovery of this fundamental property of light, which has opened the whole mystery of colours, we see our author was not only beholden to the experiments themselves, which many others had made before him, but also to his skill in geometry; which was absolutely necessary to determine what the figure of the refracted image ought to be upon the old principle of an equal refraction of all the rays: but having thus made the discovery he contrived the following experiment to prove it at sight.

II. Experiment. Newt. Opt. p. 37. Fig. 16.

In the middle of two thin boards, *DE*, *de*, I made a round hole in each, at *G* and *g*, a third part of an inch in diameter; and in the window-shut a much larger hole being made, at *F*, to let into my

darkened chamber a large beam of the ſun's light, I placed a priſm, *ABC*, behind the ſhut in that beam, to refract it towards the oppoſite wall; and cloſe behind this priſm I fixed one of the boards *DE*, in ſuch manner that the middle of the refracted light might paſs through the hole made in it at *G*, and the reſt be intercepted by the board. Then at the diſtance of about 12 feet from the firſt board I fixed the other board *de*, in ſuch manner that the middle of the refracted light, which came through the hole in the firſt board, and fell upon the oppoſite wall, might paſs through the hole *g* in this other board *de*, and the reſt being intercepted by the board might paint upon it the coloured ſpectrum of the ſun. And cloſe behind this board I fixed another priſm *abc* to refract the light which came through the hole *g*. Then I returned ſpeedily to the firſt priſm *ABC* and by turning it ſlowly to and fro about its axis, I cauſed the image which fell upon the ſecond board *de* to move up and down upon that board, that all its parts might paſs ſucceſſively through the hole in that board, and fall upon the priſm behind it. And in the mean time I noted the places, *M*, *N*, on the oppoſite wall, to which that light after its refraction in the ſecond priſm did paſs; and by the difference of the places at *M* and *N*, I found that the light, which being moſt refracted in the firſt priſm *ABC*, did go to the blue end of the image, was again more refracted by the ſecond priſm *abc*, than the light which went to the red end of that image. For when the lower part of the light which fell upon the ſecond board *de*, was caſt through the hole *g*, it went to a lower place *M* on the wall; and when the higher part of that light was caſt through the ſame hole *g*, it went to a higher place *N* on the wall; and when any intermediate part of the light was caſt through that hole, it went to ſome place in the wall between *M* and *N*. The unchanged poſition of the holes in the boards made the incidence of the rays upon the ſecond priſm to be the ſame in all caſes. And yet in that common incidence ſome of the rays were more refracted and others leſs: and thoſe were more refracted in this priſm, which by a greater refraction in the firſt priſm were more turned out of their way; and therefore for their conſtancy of being more refracted are deſervedly called more refrangible.

Our author ſhews alſo, by experiments made with a convex glaſs, that lights (reflected from natural bodies) which differ in colour, differ alſo in degrees of refrangibility[a]: and that they differ in the ſame manner as the rays of the ſun do.

[a] Newt. Opt. p. 16.

Definitions Newt. Opt. p. 4.

The light whoſe rays are all alike refrangible I call ſimple homogeneal and ſimilar, and that whoſe rays are ſome more refrangible than

than others I call compound, heterogeneal and diſſimilar. The former light I call homogeneal not becauſe I would affirm it ſo in all reſpects; but becauſe the rays which agree in refrangibility agree at leaſt in all their other properties which are conſidered in this chapter.

The colours of homogeneal lights I call primary, homogeneal and ſimple, and thoſe of heterogeneal lights, heterogeneal and compound. For theſe are always compounded of homogeneal lights, as will appear in the following articles.

The homogeneal light and rays which appear red, or rather make objects appear ſo, I call rubrifick or red-making; thoſe which make objects appear yellow, green, blue and violet, I call yellow-making, green-making, blue-making, violet-making; and ſo the reſt. And if at any time I ſpeak of light and rays as coloured or endued with colours, I would be underſtood to ſpeak not philoſophically and properly but groſly, and according to ſuch conceptions as vulgar people in ſeeing all theſe experiments would be apt to frame. For the rays to ſpeak properly are not coloured. In them there is nothing elſe than a certain power and diſpoſition to ſtir up a ſenſation of this or that colour. For as ſound in a bell or muſical ſtring or other ſounding body, is nothing but a trembling motion, and in the air nothing but that motion propagated from the object, and in the ſenſorium it is a ſenſe of that motion under the form of ſound; ſo colours in the object are nothing but a diſpoſition to reflect this or that ſort of rays more copiouſly than the reſt; in the rays they are nothing but their diſpoſitions to propagate this or that motion into the ſenſorium; and in the ſenſorium they are ſenſations of thoſe motions under the forms of colours.

Newt. Opt. p. 108.

III. Experiment. Homogeneal light is refracted regularly, &c. Newt. Opt. p. 62.

29. Homogeneal light is refracted regularly without any dilatation ſplitting or ſhattering of the rays, and the confuſed viſion of objects ſeen through refracting bodies by heterogeneal light, ariſes from the different refrangibility of ſeveral ſorts of rays. This will appear by the experiments which follow. In the middle of a black paper I made a round hole about a fifth or a ſixth part of an inch in diameter. Upon this paper I cauſed the ſpectrum of homogeneal light deſcribed in the former article, ſo to fall that ſome part of the light might paſs through the hole in the paper. This tranſmitted part of the light I refracted with a priſm placed behind the paper, and letting this refracted light fall perpendicularly upon a white paper two or three feet diſtant from the priſm, I found that the ſpectrum formed on the paper by this light was not oblong, as when it is made, in the firſt experiment, by refracting the ſun's compound

 light,

light, but was (so far as I could judge by my eye) perfectly circular, the length being no where greater than the breadth; which shews that this light is refracted regularly without any dilatation of the rays; and is an ocular demonstration of the mathematical proposition mentioned in the 28th article.

IV. Experiment. Newt. Opt. p. 63.

In the homogeneal light I placed a paper circle of a quarter of an inch in diameter; and in the sun's unrefracted, heterogeneal, white light I placed another paper circle of the same bigness; and going from these papers to the distance of some feet I viewed both circles through a prism. The circle illuminated by the sun's heterogeneal light appeared very oblong, the length being many times greater than the breadth. But the other circle illuminated with homogeneal light appeared circular and distinctly defined, as when it is viewed by the naked eye; which proves the whole proposition mentioned at the beginning of this article.

V. Experiment. Ibid.

In the homogeneal light I placed flies and such like minute objects, and viewing them through a prism I saw their parts as distinctly defined as if I had viewed them with the naked eye. The same objects placed in the sun's unrefracted heterogeneal light which was white, I viewed also through a prism, and saw them most confusedly defined, so that I could not distinguish their smaller parts from one another. I placed also the letters of a small print one while in the homogeneal light and then in the heterogeneal, and viewing them through a prism they appeared in the latter case so confused and indistinct that I could not read them; but in the former they appeared so distinct that I could read readily, and thought I saw them as distinct as when I viewed them with my naked eye; in both cases I viewed the same objects through the same prism at the same distance from me and in the same situation. There was no difference but in the lights by which the objects were illuminated and which in one case was simple in the other compound; and therefore the distinct vision in the former case and confused in the latter could arise from nothing else than from that difference in the lights. Which proves the whole proposition.

The colour of homogeneal light cannot be changed by refractions nor by reflexions. Newt. Opt. p. 107.

30. In these three experiments it is farther very remarkable that the colour of homogeneal light was never changed by the refraction: and as these colours were not changed by refractions, so neither were they by reflexions. For all white, grey, red, yellow, green, blue, violet bodies, as paper, ashes, red lead, orpiment, indigo, bise, gold, silver, copper, grass, blue flowers, violets, bubbles of water tinged with various colours, peacocks' feathers, the tincture of *lignum nephriticum* and such like, in red homogeneal light appeared totally red,

red, in blue light totally blue, in green light totally green, and ſo of other colours. In the homogeneal light of any colour they all appeared totally of that ſame colour, with this only difference, that ſome of them reflected that light more ſtrongly, others more faintly. I never yet found any body which by reflecting homogeneal light could ſenſibly change its colour.

From all which it is manifeſt, that if the ſun's light conſiſted of but one ſort of rays, there would be but one colour in the world. Nor would it be poſſible to produce any new colour by reflexions and refractions: and by conſequence that the variety of colours depends upon the compoſition of light.

The ſine of incidence of every homog. ray is to its ſine of refraction in a given ratio. Newt. Opt. p. 64. Fig. 17. [a] Art. 13.

31. Every homogeneal ray conſidered apart is refracted according to one and the ſame rule[a], ſo that its ſine of incidence is to its ſine of refraction in a given ratio: that is, every different coloured ray has a different ratio belonging to it. This our author has proved by experiment, and by other experiments has determined by what numbers thoſe given ratios are expreſſed. For inſtance, if an heterogeneal white ray of the ſun emerges out of glaſs into air, or which is the ſame thing, if rays of all colours be ſuppoſed to ſucceed one another in the ſame line *AC*, and *AD* their common ſine of incidence in glaſs be divided into 50 equal parts, then *EF* and *GH* the ſines of refraction into air, of the leaſt and moſt refrangible rays will be 77 and 78 ſuch parts reſpectively. And ſince every colour has ſeveral degrees, the ſines of refraction of all the degrees of red will have all intermediate degrees of magnitude from 77 to $77\frac{1}{8}$, of all the degrees of orange from $77\frac{1}{8}$ to $77\frac{1}{5}$, of yellow from $77\frac{1}{5}$ to $77\frac{1}{3}$, of green from $77\frac{1}{3}$ to $77\frac{1}{2}$, of blue from $77\frac{1}{2}$ to $77\frac{2}{3}$, of indigo from $77\frac{2}{3}$ to $77\frac{7}{9}$, and of violet from $77\frac{7}{9}$ to 78[b].

[b] Newt. Opt. p. 109.

The different properties of ſimple and compound colours. Newt. Opt. p. 115.

32. Colours may be produced by compoſition which ſhall be like to the colours of homogeneal light, as to the appearance of colour, but not as to the immutability of colour and conſtitution of light. And thoſe colours, by how much they are more compounded, by ſo much are they leſs full and intenſe; and by too much compoſition they may be diluted and weakened till they ceaſe, and the mixture becomes white or grey. There may be alſo colours produced by compoſition, which are not fully like any of the colours of homogeneal light. For a mixture of homogeneal red and yellow compounds an orange, like in appearance of colour to that orange which in the ſeries of unmixed priſmatick colours lyes between them. But the light of one orange is homogeneal as to refrangibility, that of the other is heterogeneal; and the colour of the one, if viewed through a priſm remains unchanged, that of the other is changed and re-

ſolved

ſolved into its component colours red and yellow. And after the ſame manner other neighbouring homogeneal colours may compound new colours, like the intermediate homogeneal ones: as yellow and green the colour between them both; and afterwards if blue be added there will be made a green, the middle colour of the three which enter the compoſition. For the yellow and blue on either hand, if they are equal in quantity, draw the intermediate green equally toward themſelves, and ſo keep it as it were in æquilibrio, that it verge not more to the yellow on one hand, than to the blue on the other, but by their mixed actions remain ſtill a middle colour. To this mixed green there may be farther added ſome red and violet, and yet the green will not preſently ceaſe but only grow leſs full and vivid; and by increaſing the red and violet, it will grow more and more dilute, until by the prevalence of the added colours it be overcome and turned into whiteneſs or ſome other colour. So if to the colour of any homogeneal light, the ſun's white light compoſed of all ſorts of rays be added, that colour will not vaniſh or change its ſpecies, but be diluted, and by adding more and more white it will be diluted more and more perpetually. Laſtly if red and violet be mingled there will be generated according to their various proportions various purples: ſuch as are not like in appearance to the colour of any homogeneal light; and of theſe purples mixed with yellow and blue may be made other new colours.

VI. Experiment. Whiteneſs may be compounded of colours. Newt. Opt. p. 117. Fig. 18.

33. Whiteneſs and all grey colours between white and black, may be compounded of colours; and the whiteneſs of the ſun's light is compounded of all the primary colours mixed in a due proportion.

For let the ſolar image *PT* fall upon a lens *MN* above four inches broad and about ſix feet diſtant from the priſm *ABC*, and ſo figured that it may cauſe the coloured light which divergeth from the priſm to converge and meet again at its focus G about 6 or 8 feet diſtant from the lens, and there to fall perpendicular upon a white paper *DE*. And if you move this paper to and fro, you will perceive that near the lens, as at *de*, the whole ſolar image, ſuppoſe at *pt*, will appear upon it intenſely coloured after the manner above explained: and that by receding from the lens thoſe colours will perpetually come towards one another, and by mixing more and more dilute one another continually, until at length the paper comes to the focus G, where by a perfect mixture they will wholly vaniſh and be converted into whiteneſs, the whole light appearing now upon the paper like a little white circle.

VII. Experiment. Newt. Opt. p. 129.

In the foregoing experiment I have produced whiteneſs by mixing the priſmatick colours. If now the colours of natural bodies are to be

be mingled, let a little water thickened with ſoap be agitated to raiſe a froth, and after that froth has ſtood a little, there will appear to one that ſhall view it intently various colours every where in the ſurface of the ſeveral bubbles, but to one that ſhall go ſo far off that he cannot diſtinguiſh the colours from one another, the whole froth will grow white with a perfect whiteneſs.

34. The colours of natural bodies ariſe from hence, that ſome of them reflect ſome ſort of rays, others other ſorts more copiouſly than the reſt. Minium reflects the leaſt refrangible or red-making rays moſt copiouſly and thence appears red. Violets reflect the moſt refrangible moſt copiouſly, and thence have their colour: and ſo of other bodies. Every body reflects the rays of its own colour more copiouſly than the reſt, and from their exceſs and predominance in the reflected light has its colour.

The permanent colours of natural bodies explained.

For if in the homogeneal lights obtained by the 2d experiment, you place bodies of ſeveral colours, you will find as I have done, that every body looks more ſplendid and luminous in the light of its own colour. Cinnaber in the homogeneal red is moſt reſplendent, in the green light it is manifeſtly leſs reſplendent, in the blue light ſtill leſs. Indigo in the violet blue light is moſt reſplendent, and its ſplendor is gradually diminiſhed as it is removed thence by degrees through the green and yellow light to the red. By a leek the green light, and next that the blue and yellow which compound green, are more ſtrongly reflected than the other colours red and violet, and ſo of the reſt. But to make theſe experiments the more manifeſt, ſuch bodies ought to be choſen as have the fulleſt and moſt vivid colours, and two of thoſe bodies are to be compared together. Thus for inſtance, if cinnaber and ultra-marine blue, or ſome other full blue be held together in the red homogeneal light, they will both appear red; but the cinnaber will appear of ſtrongly luminous and reſplendent red, and the ultra-marine blue of a faint obſcure and dark red. And if they be held together in the blue homogeneal light, they will both appear blue; but the ultra-marine will appear of a ſtrongly luminous and reſplendent blue, and the cinnaber of a faint and dark blue. Which puts it out of diſpute that the cinnaber reflects the red light much more copiouſly than the ultra-marine doth, and the ultra-marine reflects the blue light much more copiouſly than the cinnaber doth. The ſame experiment may be tried ſucceſſively with red and indigo or with any other two coloured bodies, if due allowance be made for the different ſtrength or weakneſs of their colour and light.

VIII. Experiment. Newt. Opt. p. 157.

And that this is not only a true reaſon of their colours, but even the

the only reaſon, may appear farther from this conſideration; that the colour of homogeneal light cannot be changed by the reflection of natural bodies [a]. For if bodies by reflection cannot in the leaſt change the colour of any one ſort of rays, they cannot appear coloured by any other means, than by reflecting thoſe which either are of their own colour, or by mixture muſt produce it.

[a] Art. 30.

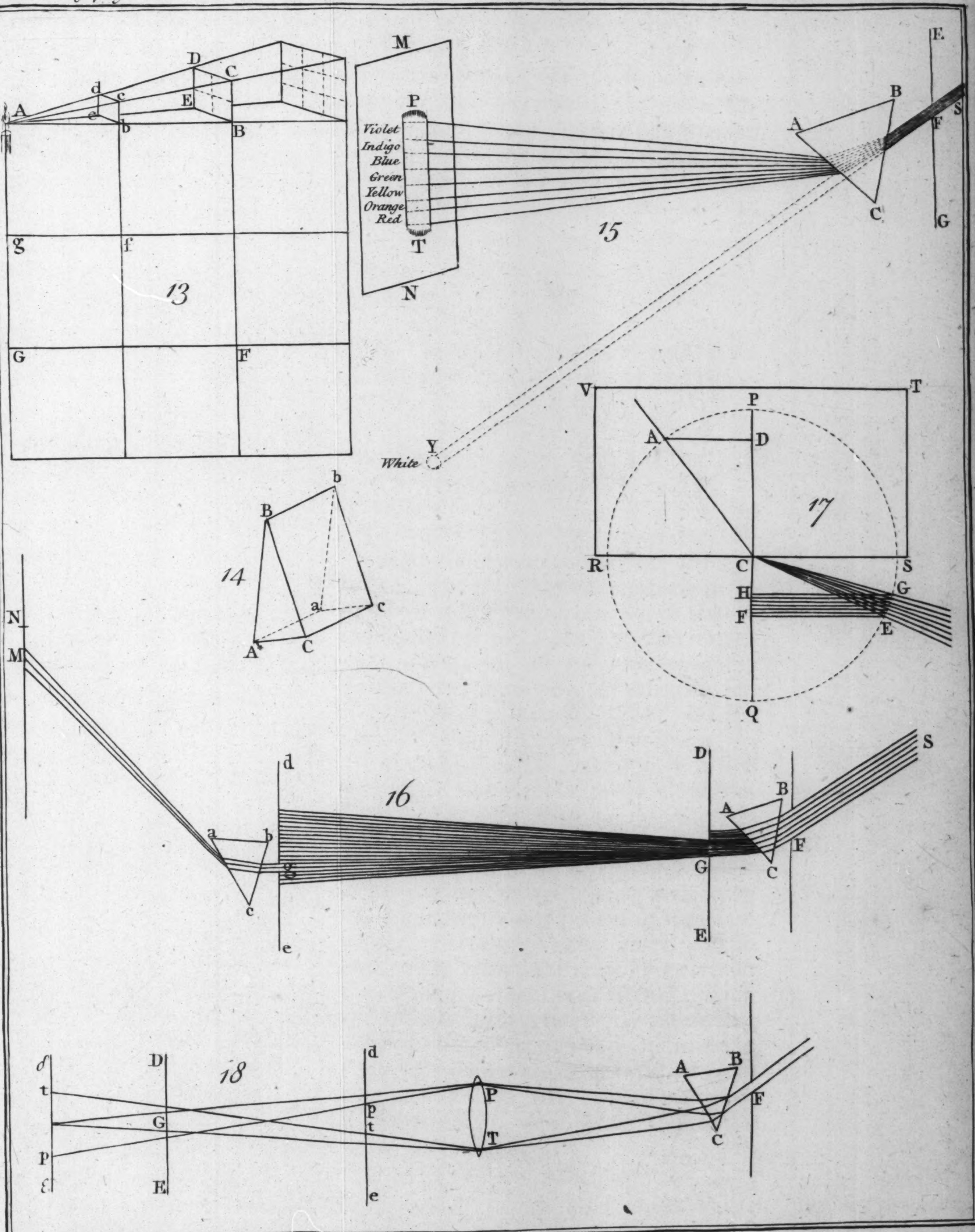
CHAP. III.

CONCERNING THE CAUSE OF REFRACTION, REFLECTION, INFLECTION AND EMISSION OF LIGHT, AND CONCERNING TRANSPARENCY, OPACITY, AND COLOURS IN BODIES.

Reflexion not cauſed by the impinging of light upon the medium. Newt. Opt. p. 237.

35. THAT the cauſe of reflection is not the impinging of light on the ſolid or impervious parts of bodies, as is commonly believed, will appear by the following conſiderations. Firſt, that in the paſſage of light out of glaſs into air there is a reflection as ſtrong as in its paſſage out of air into glaſs, or rather a little ſtronger, and by many degrees ſtronger than in its paſſage out of glaſs into water. And it ſeems not probable that air ſhould have more reflecting parts than water or glaſs. But if that ſhould poſſibly be ſuppoſed, yet it will avail nothing; for the reflection is as ſtrong or ſtronger when the air is drawn away from the glaſs, by an air-pump, as when it is adjacent to it. Secondly, if light in its paſſage out of glaſs into air be incident more obliquely than an angle of 40 or 41 degrees, it is wholly reflected, if leſs obliquely it is in a great meaſure tranſmitted [b]. Now it is not to be imagined that light at one degree of obliquity ſhould meet with pores enough in the air to tranſmit the greateſt part of it, and at another degree of obliquity ſhould meet with nothing but parts to reflect it wholly: eſpecially conſidering that in its paſſage out of air into glaſs, how oblique ſoever be its incidence, it finds pores enough in the glaſs to tranſmit a great part of it. If any man ſuppoſes that it is not reflected by the air, but by the outmoſt ſuperficial parts of the glaſs, there is ſtill the ſame difficulty: beſides that ſuch a ſuppoſition is unintelligible, and will appear to be falſe by applying water behind ſome part of the glaſs inſtead of air. For ſo in a convenient obliquity of the rays ſuppoſe of 45 or 46 degrees, at which they are all reflected where air is adjacent to the glaſs, they ſhall be in a great meaſure tranſmitted where water is adjacent to it. Which argues that their reflection or tranſmiſſion

[b] Art. 17.


13
14
15
16
17
18
Violet
Indigo
Blue
Green
Yellow
Orange
Red
White

miſſion depends on the conſtitution of air and water behind the glaſs, and not on the ſtriking of the rays on the parts of the glaſs. Thirdly, if the colours made by a priſm, placed at the entrance of a beam of light into a darkened room, be ſucceſſively caſt on a ſecond priſm placed at a great diſtance from the former, in ſuch manner that they are all alike incident upon it, (as they will be when tranſmitted through the holes in the two boards made uſe of in the 2d experiment,) the ſecond priſm may be ſo inclined to the incident rays, that thoſe which are of a blue colour ſhall be all reflected by it, and yet thoſe of a red pretty copiouſly tranſmitted. Now if reflection be cauſed by the parts of air or glaſs, I would aſk why at the ſame obliquity of incidence, the blue ſhould wholly impinge on thoſe parts, ſo as to be all reflected, and yet the red find pores enough to be in a great meaſure tranſmitted? Laſtly were the rays of light reflected by impinging on the ſolid parts of bodies, their reflections from poliſhed bodies could not be ſo regular as they are. For in poliſhing glaſs with ſand, putty or tripoli, it cannot be imagined that thoſe ſubſtances can by grating and fretting the glaſs bring all its leaſt particles to an accurate poliſh; ſo that all their ſurfaces ſhall be truly plane or truly ſpherical, and look all the ſame way, ſo as together to compoſe one even ſurface. This manner of poliſhing with powders can do no more than bring the roughneſs of the glaſs to a very fine grain, ſo that the ſcratches and frettings of the ſurface become too ſmall to be viſible. And therefore if light were reflected by impinging upon the ſolid parts of the glaſs, it would be ſcattered as much by the moſt poliſhed glaſs as by the rougheſt. So then it remains a problem how glaſs poliſhed by fretting ſubſtances can reflect light ſo regularly as it does.

Fig. 16.

36. And this problem is ſcarce otherwiſe to be ſolved than by ſaying that the reflection of a ray is effected not by a ſingle point of the reflecting body, but by ſome power of the body which is evenly diffuſed all over its ſurface, and by which it acts upon a ray without immediate contact. For that the parts of bodies do act upon light at a diſtance, will appear by the following experiments.

But by an active power diffuſed over its ſurface.

IX. Experiment. This power acts upon light at a diſtance by attracting and repelling it. Newt. Opt. p. 300.

37. The ſun ſhining into my chamber through a hole a quarter of an inch broad, I placed at the diſtance of two or three feet from the hole a ſheet of paſtboard, which was blacked all over on both ſides, and in the middle of it had a hole about three quarters of an inch ſquare for the light to paſs through. And behind the hole I faſtened to the paſtboard with pitch the blade of a ſharp knife, to intercept ſome part of the light which paſſed through the hole. The planes of the paſtboard and of the knife were parallel to one an-

other and perpendicular to the rays. And when they were ſo placed that none of the ſun's light fell upon the paſtboard, but all of it paſſed through the hole to the knife, and there part of it fell upon the blade of the knife, and part of it paſſed by its edge; I let this part of the light, which paſſed by, fall on a white paper two or three feet beyond the knife, and there ſaw two ſtreams of faint light ſhoot out both ways from the beam of light into the ſhadow, like the tails of comets. But becauſe the ſun's direct light by its brightneſs upon the paper obſcured theſe faint ſtreams ſo that I could ſcarce ſee them, I made a little hole in the midſt of the paper for that light to paſs through, and fall upon a black cloth behind it, and then I ſaw the two ſtreams plainly. They were like one another, and pretty nearly equal in length and breadth, and quantity of light. Their light at that end next the ſun's direct light was pretty ſtrong for the ſpace of about a quarter of an inch or half an inch, and in all its progreſs from that direct light decreaſed gradually till it became inſenſible. The whole length of either of theſe ſtreams meaſured upon the paper, at the diſtance of three feet from the knife, was about ſix or eight inches; ſo that it ſubtended an angle at the edge of the knife of 10 or 12, or at moſt 14 degrees.

X. Experiment. I placed another knife by this, ſo that their edges might be parallel and look towards one another, and that the beam of light might fall upon both knives and ſome part of it paſs between their edges. And when the diſtance of their edges was about the 400th part of an inch, the ſtream parted in the middle and left a ſhadow between the two parts. This ſhadow was ſo black and dark that all the light which paſſed between the two knives ſeemed to be bent and to be turned aſide to the one hand and to the other. And as the knives ſtill approached one another the ſhadow grew broader, and the ſtreams ſhorter at their inward ends next the ſhadow, until upon the contact of the knives the whole light vaniſhed and left its place to the ſhadow. And hence I gather that the light which is leaſt bent, and goes to the inward ends of the ſtreams, paſſes by the edges of the knives at the greateſt diſtance, and this diſtance when the ſhadow begins to appear between the ſtreams is about the 800th part of an inch. And the light which paſſes by the edges of the knives at diſtances ſtill leſs and leſs is more and more bent, and goes to thoſe parts of the ſtreams which are farther and farther from the direct light. Becauſe when the knives approach one another till they touch, thoſe parts of the ſtreams vaniſh laſt which are fartheſt from the direct light.

Our author has made it appear from theſe and ſome other experiments,

ments, that bodies act upon light in ſome circumſtances by an attractive and in others by a repulſive power. For he found that the ſhadows of hairs, threads, pins, ſtraws, and ſuch like ſlender ſubſtances, placed in a ſlender beam of light let into a dark room, were conſiderably broader than they ought to be, if the rays of light paſſed on by theſe bodies in right lines. Particularly he found that the ſhadow of a hair of a man's head, at the diſtance of 10 feet from the hair, was 35 times broader than the hair itſelf[a].

[a] Newt. Opt. p. 293.

38. That this power which acts upon light is infinitely ſtronger than the power of gravity will appear by the following argument. Sir *Iſaac Newton* has demonſtrated that all bodies attract one another by the force of gravity, and that the attractive forces of two homogeneal ſpheres, upon particles of matter placed very near their ſurfaces, are to each other in proportion as the diameters of the ſpheres[1]. That is to ſay, if a refracting medium be ſpherical and of the ſame denſity as the earth, the earth's force of attraction near its ſurface, will exceed the medium's force near its ſurface, as much as the diameter of the earth exceeds the diameter of the medium; or almoſt infinitely with reſpect to human conceptions. Yet we obſerve that a cannon-ball, juſt ſhot from the mouth of the cannon, is ſcarce ſenſibly deflected towards the earth by its attraction; and the leaſt particle of the ball, if it was ſeparate from the reſt, would be no more deflected than the whole; becauſe gravity makes bodies of all ſorts and ſizes deſcend with the ſame ſwiftneſs, by affecting them alike whether joined or ſeparated. Therefore a particle of light which moves, I may ſay, infinitely quicker than a cannon-ball, would be infinitely leſs bent than the particle of the ball by the attraction of the whole earth, and ſtill infinitely leſs, than this laſt bending, by the attraction of the ſpherical medium, which was ſhewn to be infinitely weaker than that of the earth. But in fact we find it is very ſenſibly bent or refracted; and therefore it muſt be affected by ſome other power of the medium, which near its ſurface is infinitely ſtronger than the power of gravity.

And is infinitely ſtronger than the power of gravity.

39. It is difficult to determine the exact law of this refractive power, or the degrees of its force at given diſtances from the refracting ſurface. However ſince we find that the effects of gravity, which decreaſe as the ſquares of the diſtances from the center increaſe, are very ſenſible at great diſtances, we may conclude that the refractive power of a medium, which at its ſurface we find is infinitely ſtronger than gravity, and yet vaniſhes at a very ſmall diſtance

And decreaſes much quicker.

1 Princip. lib. 1. prop. 74. cor. 2. & lib. 3. prop. 8.

[a] Art. 37.

from it[a], decreaſes much quicker or in a greater proportion than gravity does.

This one power both refracts and reflects light. Newt. Opt. p. 244.

[b] Art. 17.

40. It is reaſonable to conclude that bodies reflect and refract light by one and the ſame power variouſly exerciſed in various circumſtances; becauſe when light goes out of glaſs into air as obliquely as it can poſſibly do, if its incidence be made ſtill more oblique, it becomes totally reflected[b]: (for the power of the glaſs after it has reflected the light as obliquely as is poſſible, if the incidence be ſtill made more oblique becomes too ſtrong to let any of its rays go through and by conſequence cauſes total reflections:) And for this other reaſon, that thoſe ſurfaces of tranſparent bodies which have the greateſt refracting power, reflect the greateſt quantity of light, as will be ſhewn in the 47th article.

Its forces in different bodies are as their denſities nearly. Newt. Opt. p. 245.

41. From the different ratios of the ſines of incidence and refraction in a great many different bodies, our author has alſo collected that the forces of bodies to reflect and refract light are very nearly proportionable to their denſities, excepting that unctuous and ſulphureous bodies refract more than others of the ſame denſity. Whence, he ſays, it ſeems rational to attribute the refractive power of all bodies chiefly, if not wholly to the ſulphureous, oily particles with which they abound. For it is probable that all bodies abound more or leſs with ſulphurs. And as light congregated by a burning glaſs acts moſt upon ſulphureous bodies to turn them into fire and flame, ſo ſince all action is mutual, ſulphurs ought to act moſt upon light. For that the action between light and bodies is mutual, may appear from this conſideration; that the denſeſt bodies which refract and reflect light moſt ſtrongly grow hotteſt in the ſummer ſun, by the action of the refracted or reflected light. If bodies be conceived to have certain denſities exactly proportionable to their refractive powers, theſe may be called their refractive denſities.

Refractive denſities what.

This force acts in lines perpendicular to the refracting ſurface.

42. The direction of the refractive force of a medium, acting upon particles of light, is every where perpendicular to the refracting ſurface. For whether this force be a real attraction, or whether it be an impulſe upon light, cauſed by the ſpring or elaſtick power of a ſubtil fluid which pervades the medium, and being gradually denſer without than within it, may impel the light towards the medium by its greater elaſticity without than within[c]; be this as you pleaſe, yet if the medium be uniform in all its parts, its immediate power upon the light it ſelf, or upon the ſubtil fluid which acts upon it, will be equally ſtrong in every point of a plane drawn parallel to the refracting ſurface; though its ſtrength may be different in the next parallel plane, and ſtill different in the next, and ſo on as

[c] Newt. Opt. p. 323, &c.

as far as that power is extended on each ſide of the ſurface of the medium. The extent of this power will therefore be terminated by two planes, parallel to one another and to the refracting ſurface; and the ſpace between them may be called the ſpace of activity, whether the power attracts or repels. This being premiſed, I ſay the force of the medium will act upon light, either in attracting or repelling it, in lines perpendicular to its ſurface. For let *p* be a particle of light acted upon by any uniform power in the line *de* parallel to the refracting ſurface *AB*, *pc* a line perpendicular to thoſe parallels, cutting *de* in *c*; it is evident that the force of the power at *c* will move the particle *p* in the line *pc*; and taking any two points *d*, *e* at equal diſtances on each ſide of *c*, the powers at *d* and *e* being equal and acting at equal diſtances, *pd*, *pe*, equally inclined to *pc*, cannot move *p* in any direction but that of *pc*; and what has been ſaid of the equal powers in the line *de* is applicable to the powers in every line drawn parallel to *AB*, that is to the whole power of the refracting medium.

Space of activity what.

Fig. 19, 20.

43. Now when a ray of light falls perpendicularly upon the ſpace of activity its particles will be accelerated or retarded in the ſame perpendicular direction, according as the power of the medium acts with or againſt the courſe of their motion; and when the particles are got through that ſpace they will proceed with an uniform velocity. But if a ray *op* or *sr* falls obliquely upon the ſpace of activity *klmn*, the force of the medium now acting ſideways or obliquely upon the particles, will bend their courſe into a curve *pqr*, during their paſſage through that ſpace. For as light has this property in common with all other bodies, of moving ſtraight forwards, while its motion is not diſturbed by any oblique force, ſo when it is diſturbed, we may reaſonably conclude, it will follow thoſe other laws of motion, to which all other bodies are equally ſubject. The force of the medium acting ſideways upon its oblique courſe, will therefore draw it perpetually out of one direction into another. But having paſſed through the ſpace of activity, it will then proceed ſtraight forwards; for being attracted or impelled every way alike, or elſe not all if it be in empty ſpace, it will have the ſame freedom of motion in both caſes: juſt as an animal ſurrounded with air, though violently preſſed on every ſide, feels no conſtraint, but has an equal facility of moving in any direction. Thus we ſee that the refraction of light is performed in the ſame manner as if a ſtone was thrown in the direction *op*, and its courſe was bent into a curve *pqr* by its gravity; or being thrown the contrary way in the direction *sr*, it was bent into the curve *rqp* in aſcending: and ſuppoſing the

The manner of its operation in cauſing refractions and reflections. Fig. 21.

the attraction of the earth to reach no higher than the line *kl* the ſtone would from thence proceed in a ſtraight line *po*. Now the gravity of the ſtone may be ſo great, or the force of projection ſo weak, or the direction of the motion ſo oblique to the horizontal line *kl*, that it cannot aſcend ſo high as this line. In this caſe the ſtone will deſcend from the higheſt point of its courſe by the ſame degrees of curvity with which it aſcended; and if its gravity be ſuppoſed to ceaſe in all places below the line *mn*, the ſtone will go on in the direction of the laſt particle of the curve produced. This is a parallel caſe to that of reflections from the farther ſurface of denſe mediums, when the incident ray is ſo much inclined to that ſurface as to be pulled back into the ſame medium. Hitherto I have ſuppoſed the refracting medium to be contiguous to empty ſpace; but the manner of reflection and refraction is the ſame at the common ſurface of any two mediums. For ſince the ſeparate forces of the mediums act in the ſame lines, perpendicular to their common ſurface[a], and in contrary directions; the light will be affected with the difference of thoſe forces in the ſame manner as before. And if the mediums have equal forces they will balance each other, without cauſing any reflection or refraction at all. It has been obſerved already that the perpendicular breadth of the ſpace of activity is exceeding ſmall, and conſequently in phyſical experiments the incurvation of the ray may ſtill be conſidered as performed in a phyſical point.

Fig. 22.

[a] Art. 42.

And in cauſing the different refrangibility of rays. Newt. Opt. p. 347.

44. According to this theory nothing more is requiſite for producing all the variety of colours and degrees of refrangibility, than that the rays of light be bodies of different ſizes; the leaſt of which may make violet, the weakeſt and darkeſt of the colours, and be more eaſily diverted by refracting ſurfaces from its right courſe; and the reſt, as they are bigger and bigger, may make the ſtronger and more lucid colours, blue, green, yellow and red; and be more and more difficultly diverted. For particles of different ſizes, that fall upon the ſpace of activity *klmn* in the line *op*, having different forces, may deſcribe different curves, as *pa*, *pb*, *pc*, and conſequently will emerge from that ſpace in different angles.

Fig. 23.

And in cauſing the angles of reflection to be the ſame in all ſorts of rays. Fig. 23.

45. Thus may heterogeneal particles diverge from one another by refraction, though not by reflection. For if the line of their incidence *op* be ſo oblique to the ſpace of attraction *klmn*, that all the particles are pulled back into the ſame medium, they will return in parallel lines *rs*, *tv*, *xy*, &c. inclined to that ſpace in the ſame angles as the line of incidence *op* is inclined to it. Juſt as ſeveral balls of different ſizes, ſhot with different forces out of a cannon

non *op* in any fixt poſition, will deſcribe different curves, as *pdr*, *pet*, *pfx*, &c. yet in returning to the ground they will all ſtrike upon it in equal angles, at *r*, *t*, *x*, &c. every one being equal to the angle of elevation at *p*. Now ſince the ſpace of attraction is exceeding thin, the parallel lines *rs*, *tv*, &c. will be ſo cloſe together that the ſenſe cannot perceive a diſtinct ſenſation of the ſeparated particles, and conſequently the reflected and incident light will appear of the ſame colour. And when the incident light conſiſts of ſeveral rays, though the particles in each ray may be a little ſeparated after reflection, and proceed in different lines, yet thoſe ſeveral lines will be mixt together, and conſequently the reflected light will appear white or of the ſame colour as the incident light.

And in cauſing reflections from opake bodies and from the firſt ſurface of tranſparent ones.

Fig. 24.

46. Sir *Iſaac Newton*'s notion of the cauſe and manner of reflection from opake bodies, and from the firſt ſurface of tranſparent bodies, ſeems to be this that follows. Let the attractive power of the denſe medium *ABCD* end at the line *kl*, and there let the repulſive power begin[a], and let it end at the parallel line *hi*; and when a ray *op* falls from air upon the ſpace of repulſion *hikl*, it will be perpetually diverted from one direction into another by the oppoſition of the repulſive force, and ſo will deſcribe a curve *pqr*, till it emerges from that ſpace in the ſame angle at *r* with which it immerged at *p*, and then it will proceed in a right line *rs*. This will be the courſe of the ray if its progreſſive force be but weak, or the repulſive force be ſo ſtrong as to hinder it from entering the ſpace of attraction *klmn*. For if it enters this ſpace, inſtead of being reflected, it will be refracted into the denſe medium. And in reality ſome part of the incident light is always reflected and ſome refracted at all tranſparent ſurfaces; the cauſe of which our author has alſo conſidered[b].

[a] Art. 37.

[b] Newt. Opt. p. 253.

Hence it ſeems to follow that the repulſive power of a denſe medium is leſs extended or elſe weaker than the attractive. For if the bending of a ray by the repulſive power, was not leſs than the contrary bending made by the attractive, the refraction into a denſe medium could not always be made towards the perpendicular, as it always is. We may alſo obſerve that a refracted ray, in its paſſage through the ſurface of a tranſparent medium, is bent backward and forward with a motion like that of an eel; and our author takes notice of the ſame ſort of motion in its paſſage by the edges and ſides of bodies. It follows alſo that the repulſive power does not extend to a ſenſible diſtance from the medium; for if it did, it would be diſcovered by a ſenſible incurvation of the ray throughout that extent; contrary to experience.

47. Thoſe

Stronger and weaker reflections how caused.
Newt. Opt. p. 220.
[a] Art. 41.
[b] Art. 17.

47. Those superficies of transparent bodies reflect the greatest quantity of light which have the greatest refractive power; that is which intercede mediums that differ most in their refractive densities [a]: and in the confines of equally refracting mediums there is no reflection. The analogy between refraction and reflection will appear by considering, that when light passes obliquely out of one medium into another which refracts from the perpendicular, the greater is the difference of the refracting densities, the less obliquity of incidence is requisite to cause a total reflection [b]. Those superficies therefore which refract most, do soonest reflect all the light which is incident upon them, and so must be allowed most strongly reflective. But the truth of this proposition will farther appear by observing, that in the superficies interceding two transparent mediums (such as are air, water, oil, common glass, crystal, metalline glass, island glasses, white transparent arsenick, diamonds, &c.) the reflection is stronger or weaker accordingly as the superficies hath a greater or less refractive power. For in the confine of air and sal-gem it is stronger than in the confine of air and water, and still stronger in the confine of air and common glass or crystal, and stronger in the confine of air and a diamond. If any of these and such like transparent solids, be immerged in water, its reflection becomes much weaker than before, and still weaker if they be immerged in the more strongly refracting liquors of well rectified oil of vitriol or spirit of turpentine. If water be distinguished into two parts by an imaginary surface, the reflection in the confine of these two parts is none at all; in the confine of water and ice it is very little; and in that of water and oil it is something greater; in that of water and sal-gem still greater, and in that of water and glass or crystal or other denser substances still greater, accordingly as those mediums differ more or less in their refracting powers. Hence in the confine of common glass and crystal there ought to be a weak reflection, and a stronger reflection in the confine of common and metalline glass, though I have not yet tried this. But in the confine of two glasses of equal densities, as of two object-glasses of long telescopes pressed gently together, there is not any sensible reflection. For objects may be seen by rays obliquely transmitted through a round black spot where the glasses touch one another, but not through other places where the light is reflected at the interval between the glasses. And the same may be understood of the superficies interceding two crystals, or two liquors, in which no reflection is caused. So then the reason why uniform pellucid mediums, such as water, glass, or crystal, have no sensible reflection, but in their external superficies,

perficies, where they are adjacent to other mediums of a different denſity, is becauſe all their contiguous parts have one and the ſame degree of denſity: or this uniform denſity of their contiguous parts is a neceſſary condition of the tranſparency of the whole.

48. The leaſt parts of almoſt all natural bodies are in ſome meaſure tranſparent: and the opacity of thoſe bodies ariſeth from the multitude of reflections cauſed in their internal parts. That this is ſo hath been obſerved by others, and will eaſily be granted by them that have been converſant with microſcopes. And it may alſo be tried by applying any ſubſtance to a hole, through which ſome light is immitted into a dark room. For how opake ſoever that ſubſtance may ſeem in the open air, it will by that means appear very manifeſtly tranſparent, if it be of a ſufficient thinneſs. Only white metalline bodies muſt be excepted, which by reaſon of their exceſſive denſity ſeem to reflect almoſt all the light incident on their firſt ſuperficies; unleſs by ſolution in menſtruums they be reduced into very ſmall particles, and then they become tranſparent.

Opacity cauſed by a multitude of internal reflections. Newt. Opt. p. 222.

49. Between the parts of opake and coloured bodies are many ſpaces, either empty or repleniſhed with mediums of other denſities; as water between the tinging corpuſcles wherewith a liquor is impregnated; air between the aqueous globules that conſtitute clouds or miſts; and for the moſt part ſpaces void of both air and water, but yet perhaps not wholly void of all ſubſtance, between the parts of hard bodies. The truth of this is evident by the two preceding articles. For by the latter article there are many reflections made by the internal parts of bodies, which by the former article would not happen if the parts of theſe bodies were continued, without any ſuch interſtices between them: becauſe reflections are cauſed only in ſuperficies which intercede mediums of a different denſity by article 47.

The conſtitution of opake and coloured bodies what. Newt. Opt. p. 223.

But farther, that this diſcontinuity of parts is the principal cauſe of the opacity of bodies, will appear by conſidering, that opake ſubſtances become tranſparent by filling their pores with any ſubſtance of equal or almoſt equal denſities with their parts. Thus paper dipped in water or oil, the *oculus mundi* ſtone ſteeped in water, linen cloth oiled or varniſhed, and many other ſubſtances ſoaked in ſuch liquors as will intimately pervade their little pores, or ſeparating parts, become by that means more tranſparent than otherwiſe. So on the contrary, the moſt tranſparent ſubſtances may by evacuating their pores, or ſeparating parts, be rendered ſufficiently opake; as ſalts or wet paper or the *oculus mundi* ſtone by being dried; horn by being ſcraped; glaſs by being reduced to powder or otherwiſe

flawed; turpentine by being ſtirred about with water till they mix imperfectly; and water by being formed into many ſmall bubbles, either alone in the form of froth, or by ſhaking it together with oil of turpentine or oil of olive or with ſome other convenient liquor, with which it will not perfectly incorporate.

The conſtitution of tranſparent bodies what. Newt. Opt. p. 225.

50. The parts of bodies and their interſtices muſt not be leſs than of ſome definite bigneſs, to render them opake and coloured. For the opakeſt bodies, if their parts be ſubtily divided, (as metals by being diſſolved in acid menſtruums, &c.) become perfectly tranſparent; and at the ſuperficies of the object-glaſſes, mentioned in the 47th article, where they were very near to one another though they did not abſolutely touch, there was no ſenſible reflection. And likewiſe if a bubble be blown with water firſt made tenacious by diſſolving a little ſoap in it, and be covered with a clear glaſs, to defend it from being agitated by the external air, and be ſuffered to reſt a while, till by the continual ſubſiding of the water it becomes very thin; at the top where it is thinneſt, there will grow a round, black, ſpot (like that between the object-glaſſes) which will continually dilate it ſelf more and more till the bubble breaks; now this ſpot appears black and tranſparent for want of a ſenſible reflection, whereas the ſides of the bubble which are thicker than the top appear coloured and opake by a ſtrong reflection.

On theſe grounds I perceive it is that water, ſalt, glaſs, ſtones and ſuch like ſubſtances are tranſparent. For upon diverſe conſiderations they ſeem to be as full of pores or interſtices between their parts as other bodies are, but yet their parts and interſtices to be too ſmall to cauſe reflections in their common ſurfaces.

CHAP.

Chap. IV.

Concerning the Refractions of a single Ray of Light in its passage through a Prism, Globe, or Lens.

51. A RAY of light EF falling obliquely upon a flat piece of glass, or any medium terminated by two parallel planes represented by the lines AB, CD, will emerge from it after both refractions at F and G in a line GH parallel to the incident ray EF. For since any line FG which the ray describes in passing between the parallel planes, is equally inclined to them both [a], it will be bent as much at G in going forward, as it would be at F in going backward [b]; and these equal bendings being made contrary ways, the incident and emergent rays EF and GH are therefore parallel. Refraction of a ray through parallel plane surfaces. Fig. 25. [a] Eucl. I. 29. [b] Art. 11.

52. The lines described by the incident and emergent rays EF and GH, being produced are closer together when the glass is thinner, and also when the ray falls less obliquely upon it; because the bendings at F and G are then less [c]: and in these cases if the glass be not flat but bent a little as represented in the 26th figure by two parallel arches AB, CD, the line EF, GH will still be nearly parallel. For the bended surfaces refract the ray $EFGH$ just as much as two planes would do supposing they touched the surfaces at F and G [d]: and these planes will be nearly parallel when the line FG is but little inclined to the surfaces; being exactly so when it stands perpendicular to them both. Refraction of a ray through parallel spherical surfaces. [c] Art. 16. [d] Art. 19.

53. A thin piece of glass or of any transparent substance bounded on one side by a polished plane surface, represented by the line EF, and on the other side by a small portion of a polished spherical surface, represented by the arch ACB; or bounded on both sides by spherical surfaces ACB, EDF, is called a lens or simply a glass; and is conceived by mathematicians to be generated or described by turning the figure $ACBFDE$ round about the line CD, drawn through the middle of it perpendicularly to both its sides. This line CD produced is therefore called the axis of the lens; and it passes through G and H, the centers of its surfaces. The points C, D where it cuts the surfaces are called the vertexes of the lens, and the middle point between them is called its center. The 27th figure represents a plano-convex glass, the 28th a plano-concave, the 29th a double-convex, the 30th a double-concave, and the 31st and A lens what. Fig. 27. to 32.

 32d

32d two concavo-convex glaſſes, whereof the firſt is called a meniſcus, becauſe it reſembles a little moon. It muſt be remembered once for all, that the thickneſs *CD* of all theſe glaſſes is generally ſo ſmall, that it ſeldom need be conſidered [a].

[a] Art. 61.

Refractions of a ſingle ray through a priſm. Fig. 33, 34, 35.

54. When a ray of light *EFGH* is refracted at *F* and *G* in paſſing through the ſides, *AB*, *BC*, of a priſm, the courſe of the emergent ray, *GH*, always deviates from, *EF*, the courſe of the incident ray, towards the thicker part of the priſm, more or leſs, as the refracting angle *ABC* is greater or ſmaller. And if the refracting angle be given (or invariable) and the refractions be but ſmall, the quantity of deviation will alſo be given, though the poſition of the incident ray be varied at pleaſure.

Fig. 33. For ſuppoſing at firſt that the ray *FG*, within the priſm, is equally inclined to its ſides *AB*, *BC*, as in fig. 33, it is evident from the poſition of the perpendiculars to thoſe ſides at the points *F* and *G*, that both the refractions are made from the edge *B* towards the oppoſite ſide *AC* [b].

[b] Art. 12.

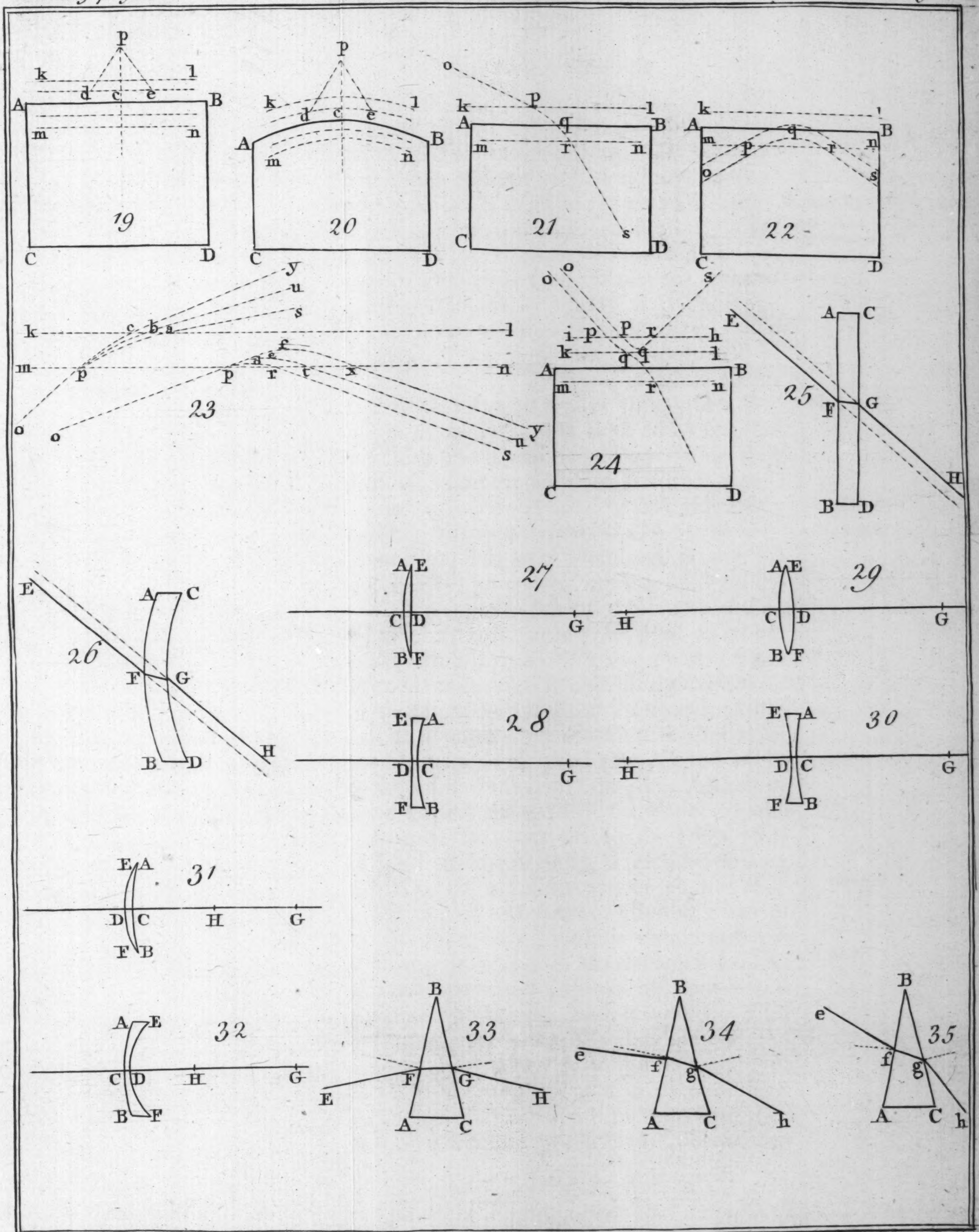
Fig. 34. Now let *FG* become unequally inclined to the ſides *AB*, *BC*, by turning it gradually into the poſition *fg*; and while it becomes leſs and leſs oblique to one ſide, ſuppoſe *AB*, it will become more and more oblique to the other ſide *BC*. Conſequently ſuppoſing a ray to go both ways along this variable line *fg*, it will be more and more bent in going through the ſide *BC* and leſs and leſs in going back through the ſide *AB*; ſo that the total bending of the ray, conſiſting of both its bendings, or angles *efg* and *fgh*, taken together, will continue to be much the ſame in all its poſitions. The circulation of the line *fg*, may be farther continued till it becomes perpendicular to the ſide *AB*; and then the bending at this ſide is nothing: it may alſo be continued ſtill farther till the bending at *f* is made the contrary way; which ſtill takes off from the perpetual increaſe of the greater bending at *g* and keeps the total bending invariable.

Fig. 35.

Fig. 34. When *fg* is perpendicular to *AB*, let the latter plane *BC* be turned gradually towards the former *BA*, upon the edge *B*, and the ray that comes along *fg* will gradually fall leſs obliquely upon it; and conſequently the bending at *g* will be gradually diminiſhed [c]; and reduced to nothing when the refracting angle *ABC* vaniſhes. Laſtly if ſeveral homogeneal rays be ſuppoſed to come parallel to one another they will all emerge parallel to one another [d]. Therefore the quantity of deviation of a ray does not at all depend upon its paſſage through a thicker or thinner part of the priſm, nor upon its inclinations to the ſides of the priſm, but is proportioned to the quantity of the refracting angle *ABC*; and the more exactly as this angle

[c] Art. 14, 15.

[d] Art. 23.


19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35

angle and the refractions at its sides are smaller. The truth of this conclusion is proved mathematically in the next article.

55. If the angles of incidence and refraction of a ray, *QACS*, Fig. 36, 37. that passes through a very small angle of a prism, *AIC*, be so little as to be reckoned proportionable to their sines; the angle of deviation *RFS*, contained under the incident ray *QAFR* and the emergent ray *SCFT* produced, will be to the refracting angle *AIC*, as the difference of the sines of incidence and refraction to the lesser of them; and consequently the magnitude of the angle of deviation *RFS* will be invariable in all positions of the ray.

For let the perpendicular *AB*, to the first surface *AI*, cross *CD*, the perpendicular to the second, in *E*; and supposing the ray *AC* to go both ways out of the prism, the angle of incidence *ACD* will be to the angle of emergence *DCT*, in the given ratio of the sine of incidence to the sine of refraction, that is of i to r; and disjointly, we have *ACD* to *ACT* as i to $r-i$; and the angle *CAB* is to *CAR*, in the same ratio, supposing the ray to go backward along *CA*; and conjointly or disjointly we have $ACD \pm CAB$ to $ACT \pm CAR$, that is *AED* or *AIC* to *RFS* in the same given ratio of i to $r-i$. *Q. E. D.*

56. *Corol.* 1. Hence any two homogeneal emergent rays produced, will be inclined to one another in the same angle as the two incident rays are inclined to one another. For let the two incident rays *QF*, *qf* Fig. 38. (produced) meet in *K*; and let the emergent rays *SF*, *sf* (produced) meet in *L*; and let one of the incident rays cross the other emergent ray in *M*; and since in the triangles *KMF*, *LMf*, the angles at *M* are equal and also those at *F* and *f* by article 55, it follows that the remaining angles at *K* and *L* are also equal.

57. *Corol.* 2. When the ray *AC* within the prism, coincides with Fig. 39. a perpendicular to either of the planes, as with *AB*; one of the refractions will vanish at *A*; and then the angle of deviation *RFS* made by the other single refraction, will continue the same in quantity as before, when it was made by two refractions; because the magnitude of the angle of deviation is invariable by article 55.

58. *Corol.* 3. Therefore when an heterogeneal ray is separated into coloured rays, by small refractions through a small refracting angle of a given quantity, the emergent rays of given colours will be inclined to one another and to the incident ray in certain given angles, in all positions of the incident ray. Because these inclinations made by two refractions, are every where equal to the inclinations made by a single refraction at the second plane, when the incident ray falls perpendicular upon the first plane.

59. When

Refractions of a single ray through the edge of a lens, or the sides of a globe.
Fig. 40. to 47.

59. When a ray of light $EFGH$ passes through the edge of a convex or concave lens, or the sides of a globe, its emergent part GH always deviates from the course of the incident part EF towards the thicker part of the glass, if the medium in which they are placed, is rarer than the globe or lens. For the refractions at F and G are the same as if they were made by two planes FA, GC, that touch the spherical surface at F and G[a]; and so the sides of the glass may be considered as inclined to each other like the sides of a prism. But the course of the ray is bent towards the thicker part of the prism[b], and therefore towards the thicker part of the globe or lens.

[a] Art. 19.
[b] Art. 54.

The 44th and 59th figures represent the refractions of a ray passing through a sphere placed within a medium denser than itself; the bending of which ray may, in like manner, be proved to be towards the thinner part of the sphere.

Refractions of a single ray through the middle of a lens.
Fig. 48. to 55.

60. From the same method of reasoning it follows, that the deviation of the course of the emergent ray from that of the incident ray is gradually diminished as the ray goes nearer and nearer to the middle of the glass; till, when it goes through the middle, its emergent and incident parts are either parallel to each other, or else are one continued line, when the ray coincides with the axis of the glass. For the angle made by the touching planes, FA, GC, is gradually diminished as the ray FG approaches to the middle: till at last it vanishes when they become parallel, as in the 51st article *.

This ray is considered as straight, and is called the axis of a pencil.

61. When a pencil of rays falls upon any glass, that ray which passes through its center, or middle point, is called the axis of the pencil. And because its incident and emergent parts EF and GH, are either one continued line or two parallel lines[c], its whole course in optical experiments may be always taken for one straight, physi-

[c] Art. 60.

* The author's demonstration of this article being applicable to those cases only, in which the angle at the vertex of the touching prism and the angles of incidence are very small[a], the following demonstration of the manner in which a ray is refracted by the sides of a *globe* is inserted from Dr. Barrow's Opt. Lect.

[a] Art. 55.

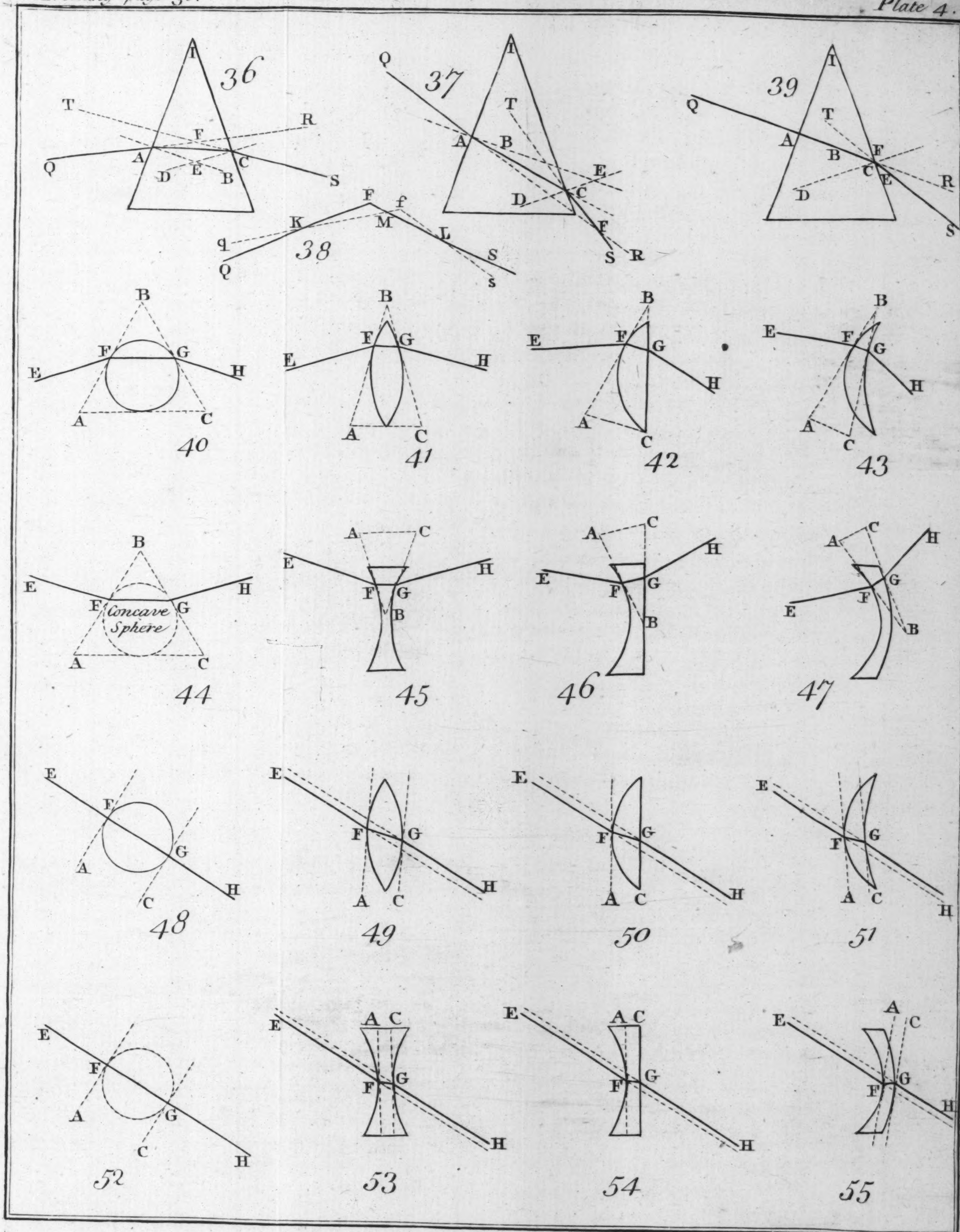
PROPOSITION. *The farther a ray is distant from the center of a globe through which it passes, the greater will be its deviation.*

Fig. 60.

Let $NRSP$ represent a globe, which differs in density from the medium in which it is placed; RS, NP, two rays passing through it, of which RS is the more remote from the center C, than NP is. Draw the radii CN, CP, CR, CS, and the angle RCS will be less than the angle NCP[b]. Therefore the sum of the angles CRS, CSR is greater than the sum of the angles CNP, CPN[c]; and each of the angles CRS, CSR is greater than each of the angles CNP, CPN[d]. But the angle of refraction at R exceeding that at N, the angle of incidence at R will also be the greater[e]. Since therefore the angles of incidence at R and S are greater than those at N and P, the deviation of RS is greater than that of NP[f]. *Q. E. D.*

[b] Euc. III. 15. & I. 25.
[c] Euc. I. 32.
[d] Euc. I. 5.
[e] Art. 13.
[f] Art. 16.

cal


36
37
38
39
40
41
42
43
Concave
Sphere
44
45
46
47
48
49
50
51
52
53
54
55

cal line: from which it differs insensibly when the thickness of the glass is small, and when the pencil falls not too obliquely upon it. Because the parallel lines EF and GH produced, go closer together in proportion as the line FG is shorter, and as the bendings at F and G are smaller.

62. When two homogeneal rays are refracted through the same point of any lens, whose thickness is inconsiderable, the angle contained under their incident parts is equal to the angle under their emergent parts. For the thickness of the lens being very small, if the rays have a common point of incidence, their points of emergence will be very near one another; or if the point of emergence be common to both rays, their points of incidence will be very near one another; and still nearer if the rays cross one another within the lens; consequently the refractions through the lens will be nearly the same as through two planes, that touch its surfaces at two given points, near the points of incidence and emergence, and contain a given angle with one another[a]. Refractions of two homogeneal rays through the same point of any lens. Fig. 57, 58.

[a] Art. 56.

63. An heterogeneal ray, refracted through a given point in a lens, has the same property as in the prism; that is, the emergent rays of given colours are inclined to one another and to the incident ray in given angles in all positions; for the reasons mentioned in the last article and in article 58. Refractions of an heterogeneal ray through the same point in any lens.

64. All rays, as $EFGH$ and $efgh$, which cross each other in a refracting globe, and pass through it at equal distances from its center, so as to touch a concentrick globe, are equally bent. For in this case the chords FG, fg being equal, their obliquities to the surface of the globe are also equal, and consequently the bendings of the ray $EFGH$ at F and G, both severally and together, are equal to the bendings of the ray $efgh$ at f and g: as is evident by conceiving the rays to go both ways along the chords FG, fg. Therefore the angle made by the incident and emergent parts of one ray, produced till they meet, will be equal to the angle made by the incident and emergent parts of the other ray produced till they meet; which is what I mean when I say the rays are equally bent. Rays are equally bent which pass at equal distances from the center of a globe. Fig. 56. to 59.

65. All rays, as $EFGH$, $efgh$, which cross each other at any given point of a lens, or which pass through it at equal distances from its center, are equally bent, provided they do not fall very obliquely upon it. Imagine a line FG within the glass, at first to be equally inclined to its sides, and then to be turned a little about any point of it, till it comes into the position fg; and while it becomes more and more oblique to one side of the glass, suppose Ff, it will become And from the center of a lens. Fig. 57. 58.

become leſs and leſs oblique to the other ſide *G g*. Conſequently if a ray be ſuppoſed to go both ways along this variable line *f g* it will be more and more bent in going through the ſide *F f*, and leſs and leſs bent in going through the other ſide *G g* [a]: ſo that the total bending of the ray conſiſting of both its bendings, or angles *e f g*, *f g h*, taken together, will continue to be much the ſame in all its poſitions. The circulation of the line *f g*, about the given point, may be farther continued till the bending at *g* is diminiſhed to nothing; and ſtill farther till it be made the contrary way; (as was explained in the 54th article;) which ſtill takes off from the perpetual increaſe of the greater bending at *f* and keeps the total invariable. To keep the ſame bending it is only neceſſary that the rays *FG*, *f g* ſhould keep at equal diſtance from the axis of the lens as near as poſſible: and nothing alters the total bending but the alteration of that diſtance [b]; becauſe the inclination of the tangent planes, like the refracting angle of a priſm, will then only be altered.

[a] Art. 16.
[b] Art. 54.

CHAP. V.

TO FIND THE FOCUS OF RAYS REFLECTED FROM ANY GIVEN SURFACE.

PROPOSITION I.

Fig. 61. 66. LET ACB *be a reflecting plane, and* Q *the focus of the incident rays, and* QC *a perpendicular to that plane; and if this perpendicular be produced to* q, *ſo that* qC *be equal to* QC, *the point* q *ſhall be the focus of the reflected rays.*

For let *QA* be any incident ray; draw *qA* and produce it towards *O*, and *CA* towards *D*. Then becauſe *Cq* is made equal to *CQ*, the triangles *CAq*, *CAQ* will be equal [a]. And conſequently the angle *DAO*, which is equal to the oppoſite angle *CAq*, is alſo equal to the angle *CAQ*. Therefore *AO* is the reflected ray [b]. *Q. E. D.*

67. *Corol.* Hence all the rays that flow towards *q*, will flow to *Q* after reflection [c].

[a] 8th Ax. Euc.
[b] Art. 9.
[c] Art. 11.

LEMMA.

68. *Quantities and their proportions, which ſo approach to a ſtate of equality as to become equal at laſt, may be taken for equal in a ſtate immediately*

diately preceding the last; and also in a state somewhat remote from the last without sensible error in physical subjects: and the same may be said of figures which continually approach to a state of similitude; especially if these errors, when computed, are found inconsiderable.

The meaning of the lemma will appear very plain when applied to the following propositions.

PROPOSITION II.

69. *When parallel rays as* DA, EC *fall almost perpendicularly upon a spherical surface* ACB, *the focus*, T, *of the reflected rays will bisect that semidiameter* EC, *which is parallel to the incident rays.* Fig. 62, 63.

For drawing EA, it will be perpendicular to the spherical surface at A, and since EC is in the same plane as the angle of incidence DAE, the reflected ray Aq (produced backwards in fig. 63.) will meet EC somewhere in q[a]; so that the angle of reflection EAq may be equal[b] to the angle of incidence EAD, or to the alternate angle AEq. The two sides Aq, Eq of the triangle AqE are therefore equal to each other[c]; and consequently each of them greater than half the third side EA, or than ET by construction. But as the point of incidence at A approaches towards C, the lines Eq, ET, continually approach towards equality, and become equal when the triangle AEq is vanishing: and so the focus of rays falling almost perpendicularly on the surface, or the nearest to the point C, is to be reckoned at T[d]. *Q. E. D.*

[a] Art. 7.
[b] Art. 8.
[c] Euc. I. 6.
[d] Art. 68.

70. *Corol.* Hence if T be the focus of the incident rays, the reflected ones will go parallel to the line TE[e].

[e] Art. 11.

The point T is called the principal focus of the reflecter ACB, and TE its focal distance. And in general, when the rays come parallel to each other, that point to which they converge or from which they diverge after reflection or refraction, is called the *principal focus* of the reflecter or refracter. And the distance of that point from the center of the reflecter or refracter is called the *focal distance* of the reflecter or refracter, and by some authors its *focal length*; in figures 89, 90, 95, &c. to 100, F is the principal focus, and FE the focal distance.

PROPOSITION III.

71. *Let* ACB *be a reflecting surface of any sphere whose center is* E. *Bisect any radius thereof, suppose* EC, *in* T; *and if in this radius, on the same side of the point* T, *you take the points* Q *and* q, *so that* TQ, TE *and* Tq *be continual proportionals; and the point* Q *be the focus of the incident rays, the point* q *shall be the focus of the reflected ones.* Fig. 64. 65.

Let QA and Aq be an incident and reflected ray (produced) making equal angles with the perpendicular AE; and the reflected ray Aq (produced) will cut QE (produced) somewhere in q, as being in the plane of incidence EAQ[a]. Draw EG parallel to Aq, and let it meet AQ in G; and also Eg parallel to AQ, meeting Aq in g: then because the angles EAG, EAg are equal[b], it follows that the triangles EAG, EAg are equiangular at their common base AE[c], and therefore equicrural[d]; and also equal to each other; and consequently each side of the equilateral figure $AGEg$, in its vanishing state when A comes to C, will be equal to half its diagonal AE[e], or by construction to ET. Now because the triangles GQE, gEq are equiangular[f], it will be as GQ to GE, so gE to gq[g]; that is, when the point A is coinciding with C, and consequently the points G, g with T, as TQ to TE so TE to Tq[h]. Q.E.D.

[a] Art. 7. [b] Art. 8. [c] Euc. I. 29. [d] Euc. I. 6. [e] Art. 68. [f] Euc. I. 29. [g] Euc. VI. 4. [h] Art. 68.

72. *Corol.* 1. If q be the focus of incident rays, Q will be the focus of the reflected ones[i]. And if either of these focuses recedes from T, the other will approach towards it. For the middle term TE of the continual proportionals TQ, TE, Tq[k], being invariable in the same reflecter, the rectangle under the extremes is also invariable[l]; and therefore TQ varies as Tq inversely. These focuses will meet each other at the center E, and at the surface C. For at these points, the three terms in the proportion are equal[m].

[i] Art. 11. [k] Art. 71. [l] Euc. VI. 17. [m] Art. 69.

73. *Corol.* 2. The rays that belong to Q may be reckoned parallel when the distance TQ is infinite, and then by this proposition its reciprocal Tq becomes nothing; which is the second proposition.

Fig. 65. 74. *Corol.* 3. Hence also we may deduce the first proposition; for supposing Q the focus of incident rays upon the convex surface AB; since TQ, TC, Tq are continual proportionals, it is well known that their differences CQ, Cq must become equal when the lines themselves are infinitely great; that is when the surface becomes a plane by removing its center to an infinite distance.

The figures serve for the cases of a convex surface supposing the incident rays to go backwards in the same lines produced through the surface.

75. By the demonstrations of the two last propositions, it appears that the focus of reflected rays there determined, is nothing else in strictness of geometry, but the intersection of the axis of the surface, that is of the ray passing through its center, and of the nearest rays to it: and also that other rays intersect the axis in different points farther and farther from that focus, as they fall farther and farther from the vertex of the surface. So that a spherical surface cannot possibly reflect all the incident rays to a single point.

point *. Neverthelefs when thefe aberrations of the remoter rays from the geometrical focus fhall be confidered, it will appear hereafter, that the denfity of their interfections, near that focus, is immenfely greater than their denfity at any confiderable diftance from it. So that in phyfical things, the focus of all the rays, that fall almoft perpendicular upon a fpherical furface, may be confidered as a phyfical point. And the fame is to be underftood of the focus of refracted rays, as will appear by the like fort of demonftrations [a]. [a] Chap. XI.

76. Hence it appears that the focus of rays reflected from any curved furface whatever, muft be reckoned the fame as if they were reflected from a fpherical furface of an equal curvity to that furface about the points of incidence. As if CD be any curve whatever, C Fig. 66. the point of incidence, $CE\mathfrak{Q}$ perpendicular to the curve, or to its tangent at C, CE the radius of a circle ACB of the fame degree of curvity at C; the rays coming parallel to CE, will be reflected to the fame focus T from either of the furfaces; and alfo the rays that flow from any point $\mathfrak{Q}$, will be reflected by either furface to the fame focus q. Becaufe we confider the focus of thofe rays only, that fall

* All the incident rays belonging to the fame pencil may be reflected to a fingle point by means of furfaces, which are generated by the revolution of fome conic fection round its tranfverfe axis. For

1ft, Let AC reprefent a parabola, the axis of which is $C\mathfrak{Q}$ and focus T; defcribe a Fig. 67. paraboloid ACB by turning AC round its axis; draw EF touching its furface in any point A; draw alfo the diameter dAD, join AT and produce it to t. Now the angles DAF, TAE are equal [a], and therefore the angle dAE equals tAF. Since then the incident and reflected rays are equally inclined to the reflecting plane [b], if DA and dA be incident rays, AT and At will be their refpective reflected rays; or if TA, tA be incident rays, AD, Ad will be their refpective reflected rays. [a] Hamilton's Conics, II. 15. [b] Art. 9.

Hence, all the rays which are incident on the concave fide of this paraboloid parallel to its axis will be reflected converging to the focus: and all the rays, which diverge from the focus will be reflected parallel to the axis. The convex furface will make parallel rays diverge from a fingle point, and converging rays go parallel, after reflection.

2dly, Let $AC\mathfrak{Q}$ reprefent an ellipfe, and ACB an hyperbola, the tranfverfe axis of Fig. 68, 69. which is $\mathfrak{Q}C$, and T, D, the foci; let a fpheroid be generated by turning the ellipfe $\mathfrak{Q}AC$ round $\mathfrak{Q}C$, and an hyperboloid by the motion of AC round its tranfverfe axis; draw EF touching the furface in any point A; join TA, DA, and produce them to t and d. Since the angles, which TAt, DAd, make with FE are equal [c], and fince the incident and reflected rays are equally inclined to the reflecting plane [b], it is evident that, if TA or tA be an incident ray, Ad or AD will be the direction in which they are reflected. [c] Hamilton's Conics, II. 16, & 17.

Hence all the rays, which are incident diverging from one focus of the fpheroid, will be reflected converging to the other focus; and all the rays, which are incident upon the convex fide of the fpheroid converging to one focus, will be reflected diverging from the other. In like manner, the convex fide of the hyperboloid will make a pencil of diverging rays diverge from, and a pencil of converging rays converge to, a fingle point after reflection. It will make diverging rays diverge more, and converging rays converge lefs. And the concave fide of this reflecter increafes the convergency, and leffens the divergency, of a pencil of rays.

upon the common points of both curves about *C*, all the rest being dispersed much thinner into other places.

77. In all these propositions when the focuses *Q*, *q* lye on the same side of the reflecting surface, if the incident rays flow from *Q* the reflected ones will flow towards *q*; and if the incident rays flow towards *Q*, the reflected ones will flow from *q*; and the contrary happens when *Q* and *q* are on contrary sides of the surface. Because the incident and reflected rays go contrary ways.

CHAP. VI.

TO DETERMINE THE PLACE, MAGNITUDE AND SITUATION OF IMAGES FORMED BY REFLECTED RAYS.

PROPOSITION I.

78. IMAGES *formed by reflections from a plane surface are similar and equal to the objects; and their parts have the same situation with respect to the backside of the plane as the parts of the object have with respect to its foreside.*

Fig. 70, 71. From any number of points *P*, *Q*, *R* of an object in any situation, draw the perpendiculars *PA*, *QC*, *RB* to the plane *ACB*, and produce them through it to the points *p*, *q*, *r*, each as far behind the plane as *P*, *Q*, *R* are before it. The points *p*, *q*, *r* being the respective focuses of the rays that belonged to *P*, *Q*, *R*[a], and being evidently in the same order, together with infinite others, will constitute an image of the object, equal to it in the whole and in every corresponding part, and alike situated: as will appear by conceiving the surface of the object, and of its image, divided into corresponding lines, such as *PQR*, *pqr*, by planes such as *PprR* perpendicular to the reflecting plane; and by folding up or doubling each plane in its line of intersection, *AB*, with the reflecting plane. For the parts of each plane on each side of *AB* will exactly cover each other, as appears by the construction. *Q. E. D.*

[a] Art. 66.

PROPOSITION II.

Fig. 72. to 75. 79. *If an arch of a circle* PQR, *concentrick to a concave or convex spherical surface* AB, *be considered as an object, its image* pqr *will also be a similar concentrick arch, whose length will be to the length of the object, in the ratio of their distances from the common center* E; *and its situation will be erect or inverted, according as it is on the same or the opposite side of the center to the object.*

For

For as the focus Q was found by making TQ, TE, Tq continual proportionals in the line QE drawn through the center[a]; so the focus p, of rays that belong to any other point P, is found by drawing PEA, and bisecting EA in S, and by making SP, SE, Sp continual proportionals. The two first terms of one proportion are severally equal to the two first of the other; and consequently the third terms Tq, Sp are equal; and thence Ep and Eq are equal. The same being true of every point of the circular object PQR, shews that its image pqr is a concentrick arch, similar to it, both being terminated by the same lines EPp, ERr; and consequently their lengths are in the same ratio as their semidiameters EQ, Eq. Lastly, according as the corresponding extremities P and p, of the object and image, are on the same or opposite sides of the center E, they are also on the same or opposite sides of their middle points Q, q; that is, the image is accordingly erect or inverted. *Q. E. D.*

[a] Art. 71.

80. *Corol.* The smaller the circular object is with respect to its radius or distance from the center, the nearer it approaches in shape to a straight line, and so does its similar image. Consequently a small straight object, placed at a good distance from the center of the glass, may be reckoned to have a straight image very nearly: though in strictness of geometry it is an arch of a conick section.

81. The images of all sorts of objects may be determined, by finding the images of their out-lines, by the foregoing propositions. For instance, if the plane of the figures PER, pEr be turned round their common diameter QEq, the circular surface generated by pqr will be the image of the circular object generated by PQR: and if the same figures PER, pEr be moved a little about an axis EF, situated in their own plane, and perpendicular to the diameter QEq, the curvilinear figure generated by this motion of pqr, will be the image of a similar figure generated by PQR. Because the reflecting arch ACB generates the reflecting spherical surface at the same time.

82. But if the whole figure $PERrp$ be moved parallel to itself in a direction EF, now perpendicular to its own plane, so that the arch ACB may generate a portion of a cylindrical surface, the figure described by this motion of pqr, will still be the image of that described by PQR; but will not be similar to it, except when they are placed at equal distances on each side the center E, and consequently are equal to each other: and their dissimilitude will be so much the greater as the disproportion between Eq and EQ, or between their lengths pr, PR, is greater; their breadths, described by the motion aforesaid, being always equal to each other.

CHAP.

CHAP. VII.

TO FIND THE FOCUS OF RAYS FALLING ALMOST PERPENDICULARLY UPON ANY REFRACTING SURFACE, SPHERE OR LENS.

DEFINITION.

Fig. 76, 77. 83. THE ſine of an angle *ABC*, or of an arch *AC* that meaſures that angle, is a line *AD* drawn from the extremity of one of the ſemidiameters, *AB*, *BC*, perpendicular to the other, produced if the angle be obtuſe. And therefore an angle *ABC* and its complement *ABE*, to two right angles, have each the ſame ſine *AD*; and when the ſines of ſeveral angles are compared together, they are always underſtood to belong to the ſame or to equal circles.

84. The ſines of very ſmall angles, and of their complements, become at laſt inſenſibly different from the arches that meaſure them; and conſequently are proportionable to the angles themſelves.

LEMMA.

Fig. 78. 85. *The ſines of the angles of any triangle are proportionable to the oppoſite ſides: as in the triangle* ABC, *the ſine of the angle* ABC *is to the ſine of the angle* BCA, *as* CA *to* AB.

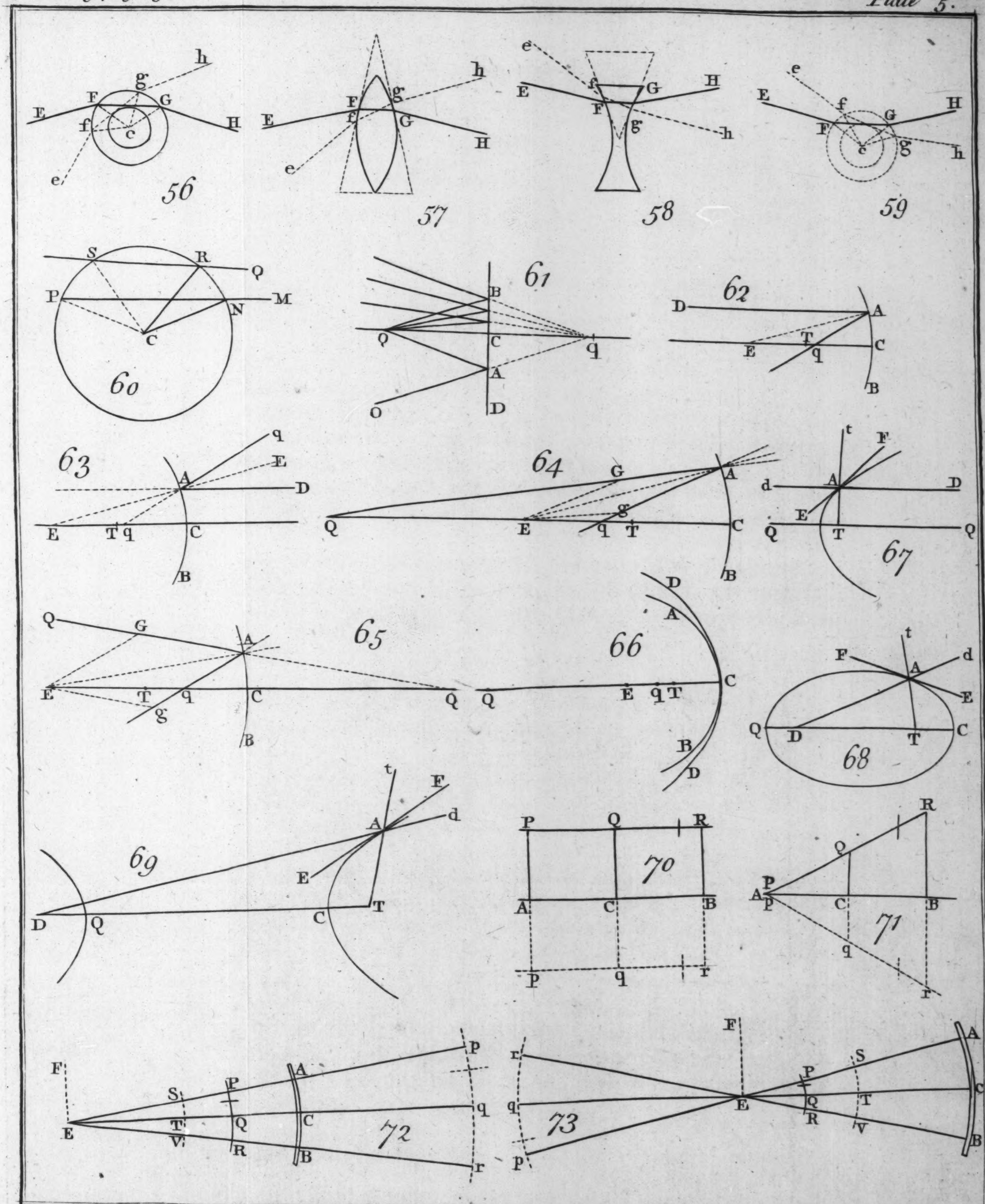
For the perpendiculars *CD*, *BE* upon thoſe ſides *AB*, *AC* produced, are the ſines of the angles *ABC*, *BCA* or *BCE* with reſpect
[a] Art. 83. to circles whoſe radius is *BC*[a]. And ſince the triangles *CAD*, *BAE*
[b] Euc. I. 32. Cor. 2. are equiangular[b], we have *CD* to *BE* as *CA* to *AB*. *Q. E. D.*

86. *Corol.* Small angles, as *BAC*, *BCE*, ſubtended by the ſame perpendicular *BE*, are reciprocally as their legs *BA*, *BC* or *EA*,
[c] Art. 84. *EC*. For the angle *BAC* is to *BCE*, when very ſmall, as the ſine
[d] Art. 85. of *BAC* to the ſine of *BCE*[c], or as *BC* to *BA*[d], or as *EC* to *EA*[e].
[e] Art. 68.

PROPOSITION I.

Fig. 79. to 82. 87. *Let* ACB *be a refracting plane, and* Q *the focus of the incident rays, and* QC *a perpendicular to that plane; and if* qC *be taken in this perpendicular, on the ſame ſide of the plane as* QC, *and in proportion to* QC, *as the ſine of incidence to the ſine of refraction, the point* q *ſhall be the focus of the refracted rays.*

For


56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73

For let the lines QA and Aq, produced as in the figures, represent an incident and a refracted ray, cutting QC in any point q whatever; and since a perpendicular to the plane at A is parallel to QC, the angle AQC will be equal to the angle of incidence, and AqC to the angle of refraction. Therefore since equal angles have equal sines, the sine of incidence is to the sine of refraction (as the sine of the angle AQC to the sine of AqC, or as Aq to AQ[a], that is when the ray QA is almost perpendicular to the plane AB) as Cq to CQ[b]. Q. E. D. [a] Art. 85. [b] Art. 68.

88. *Cor.* 1. If the surface ACB is glass, qQ is half of QC, and a third of qC. For qC is to QC as 3 to 2[c]; and therefore qQ is to QC as 1 to 2[d]. In like manner, if ACB is water, qQ is a third of QC, and a fourth of qC. [c] Art. 13. [d] Euc. V. Def. 16.

89. *Cor.* 2. Hence the refractions of a pencil of rays through a glass prism may be determined. Let Q be the focus of incident rays upon its first side AB, and QC perpendicular to AB. To QC add QT, equal to half QC; and T will be the focus of the rays QA, QB, &c. after refraction at the surface AB[e]; and being also the focus of incident rays at a and b upon the second surface ab, from Tc, drawn perpendicular to ab, take away Tq equal to a third part of Tc[e]; and q will be the focus of the emergent rays qa, qb produced. Fig. 83. [e] Art. 88.

90. *Cor.* 3. Hence the focuses of incident and emergent rays at a prism lye always very nearly at equal distances from it; provided the refractions and the refracting angle be but small. For then the perpendiculars TC, Tc are nearly equal; and in glass QC and qc are two thirds of them respectively.

91. *Cor.* 4. By proceeding in the same manner as in the 2d Coroll. it is found that, when the planes AB, ab are parallel, and TC, Tc coincide, Qq is one third of Cc, the thickness of the glass *. Fig. 83, 84.

* But the distance Cc, between the parallel planes, and the ratio between the sines of incidence and refraction at the first surface, I and R, being given, the distance Qq may be found more readily in all cases by means of the following proposition.

When a pencil of converging or diverging rays passes through two parallel planes, the distance between the planes is to the distance between the foci of incident and emergent rays, as the sine of incidence on the first surface is to the difference between the sines of incidence and refraction.

Let AB, ab, be the refracting planes; Q the focus of incident rays; QCc a perpendicular to the planes; T the point in this perpendicular where the directions of the refracted rays Aa, Bb, meet it; q the focus of emergent rays. And QA being parallel to the base of the triangle Tqa[a], Fig. 84. [a] Art. 51.

	$TQ : Qq :: TA : Aa$[b], that is, as $TC : Cc$[b] [b] Euc. VI. 2.
And by alt. and inver.	$TC : TQ :: Cc : Qq$
But	$TC : QC :: I : R$[c] [c] Art. 87.
And by conver.	$TC : TQ :: I : I-R$
Therefore	$Cc : Qq :: I : I-R$. Q. E. D.

PRO-

PROPOSITION II.

Fig. 85. to 88. 92. *Let* ACB *represent a refracting spherical surface whose center is* E, *and let the incident rays as* DA *come parallel to any semidiameter* CE, *in which produced forward or backward, according as the denser medium is convex or concave, take* CT *to* CE *as the sine of incidence to the difference of the sines, and* T *will be the focus of the refracted rays.*

For let the refracted ray *AT* (produced) cut the semidiameter *CE* produced, in any point *T* whatever; and since the semidiameter *EA* is perpendicular to the refracting surface at *A*, the angle of incidence will be equal to the angle *AEC*, and the angle of refraction, or its complement to two right ones, will be *EAT*; consequently the sine of incidence is to the sine of refraction, (as the sine of the angle *AEC*, to the sine of *EAT*, or as *AT* to *TE* [a], that is, when *A* comes nearest to *C*, and so the incident rays are almost perpendicular to the surface,) as *CT* to *TE* [b]; and disjointly [c] the sine of incidence is to the difference of the sines as *CT* to *CE*. *Q. E. D.*

[a] Art. 85. [b] Art. 68. [c] Euc. V. 17.

93. *Corol.* 1. *CT* is to *TE* as the sine of incidence to the sine of refraction.

94. *Corol.* 2. If this point *T* be the focus of incident rays, the refracted rays will go parallel to *TE* [d].

[d] Art. 11.

PROPOSITION III.

Fig. 89, 90. 95. *When parallel rays fall upon a sphere, either denser or rarer than the ambient medium, in the diameter* CD *produced, which is parallel to the incident rays, as* QA, *let* T *be their focus after their first refraction at* AC; *and the point* F *which bisects* TD *shall be their focus after their second refraction at* DG.

For let the incident and emergent rays, *QA*, *FG* produced, meet in *H*, and since the refractions at *A* and *G* are equal, as appears by supposing a ray to go both ways along the chord *AG*, the triangle *AHG* is equiangular at its base *AG*, and therefore equicrural [e]; and so is the similar triangle *GFT*, the lines *AH*, *FT* being parallel. Therefore when *A* approaches toward *C* till *G* is coinciding with *D* and the triangle *GFT* is vanishing, the leg *GF* will become equal to half the base *GT*; that is, *DF* will become equal to half *DT* [f]. *Q. E. D.*

[e] Euc. I. 6. [f] Art. 68.

LEMMA.

Fig. 91. to 94. 96. *There is a certain point* E *within every double convex or double concave lens, through which every ray that passes, will have its incident and emergent*

emergent parts QA, aq *parallel to each other: but in a plano-convex or plano-concave lens that point* E *is removed to the vertex of the concave or convex surface; and in a meniscus and in that other concavo-convex lens, it is removed a little way out of them, and lyes next to the surface which has the greatest curvity.*

For let *REr* be the axis of the lens joining the centers *R*, *r* of its surfaces *A*, *a*. Draw any two of their semidiameters *RA*, *ra* parallel to each other, and join the points *A*, *a*, and the line *Aa* will cut the axis in the point *E* above described. For the triangles *REA*, *rEa* being equiangular, *RE* will be to *Er* in the given ratio of the semidiameters *RA*, *ra*; and consequently the point *E* is invariable in the same lens. Now supposing a ray to pass both ways along the line *Aa*, it being equally inclined to the perpendiculars to the surfaces, will be equally bent and contrary ways in going out of the lens; so that its emergent parts *AQ*, *aq* will be parallel. Now any of these lenses will become plano-convex or plano-concave, by conceiving one of the semidiameters *RA*, *ra* to become infinite, and consequently to become parallel to the axis of the lens, and then the other semidiameter will coincide with the axis; and so the points *A*, *E* or *a*, *E* will coincide. *Q. E. D.*

97. *Corol.* Hence when a pencil of rays falls almost perpendicularly upon any lens, whose thickness is inconsiderable, the course of the ray which passes through *E*, above described, may be taken for a straight line passing through the center of the lens, without sensible error in sensible things. For it is manifest from the length of *Aa* and from the quantity of the refractions at its extremities, that the perpendicular distance of *AQ*, *aq* when produced, will be diminished both as the thickness of the lens and the obliquity of the ray is diminished.

DEFINITION. The point *E* is called the center of the lens.

PROPOSITION IV.

98. *To find the focus of parallel rays falling almost perpendicularly upon any given lens.* Fig. 95. to 100.

Let *E* be the center of the lens, *R* and *r* the centers of its surfaces, *Rr* its axis, *gEG* a line parallel to the incident rays upon the surface *B*, whose center is *R*. Parallel to *gE* draw a semidiameter *BR*, in which produced let *V* be the focus of the rays after their first refraction at the surface *B*, and joining *Vr* let it cut *gE* produced in *G*, and *G* will be the focus of the rays that emerge from the lens.

For since *V* is also the focus of the rays incident upon the second

 surface

surface A, the emergent rays must have their focus in some point of that ray which passes straight through this surface; that is in the line Vr, drawn through its center r: and since the whole course of another ray is reckoned a straight line gEG[a], its intersection G with Vr determines the focus of them all. $\mathcal{Q}. E. D.$

[a] Art. 97.

99. *Corol.* 1. When the incident rays are parallel to the axis rR, the focal distance EF is equal to EG. For let the incident rays that were parallel to gE be gradually more inclined to the axis till they become parallel to it; and their first and second focuses V and G will describe circular arches VT and GF whose centers are R and E. For the line RV is invariable; being in proportion to RB in a given ratio of the lesser of the sines of incidence and refraction to their difference[b]; consequently the line EG is also invariable, being in proportion to the given line RV in the given ratio of rE to rR, because the triangles EGr, RVr are equiangular.

[b] Art. 92.

100. *Corol.* 2. The last proportion gives the following rule for finding the focal distance of any thin lens. As Rr, the interval between the centers of the surfaces, is to rE, the semidiameter of the second surface, so is RV or RT, the continuation of the first semidiameter to the first focus, to EG or EF, the focal distance of the lens. Which according as the lens is thicker or thinner in the middle than at its edges, must lye on the same side as the emergent rays or the opposite side.

101. *Corol.* 3. Hence when rays fall parallel on both sides of any lens, the focal distances EF, Ef are equal. For let rt be the continuation of the semidiameter Er to the first focus t of rays falling parallel upon the surface A; and the same rule that gave rR to rE as RT to EF, gives also rR to RE as rt to Ef. Whence Ef and EF are equal, because the rectangles under rE, RT and also under RE, rt are equal. For rE is to rt and also RE to RT in the same given ratio[c].

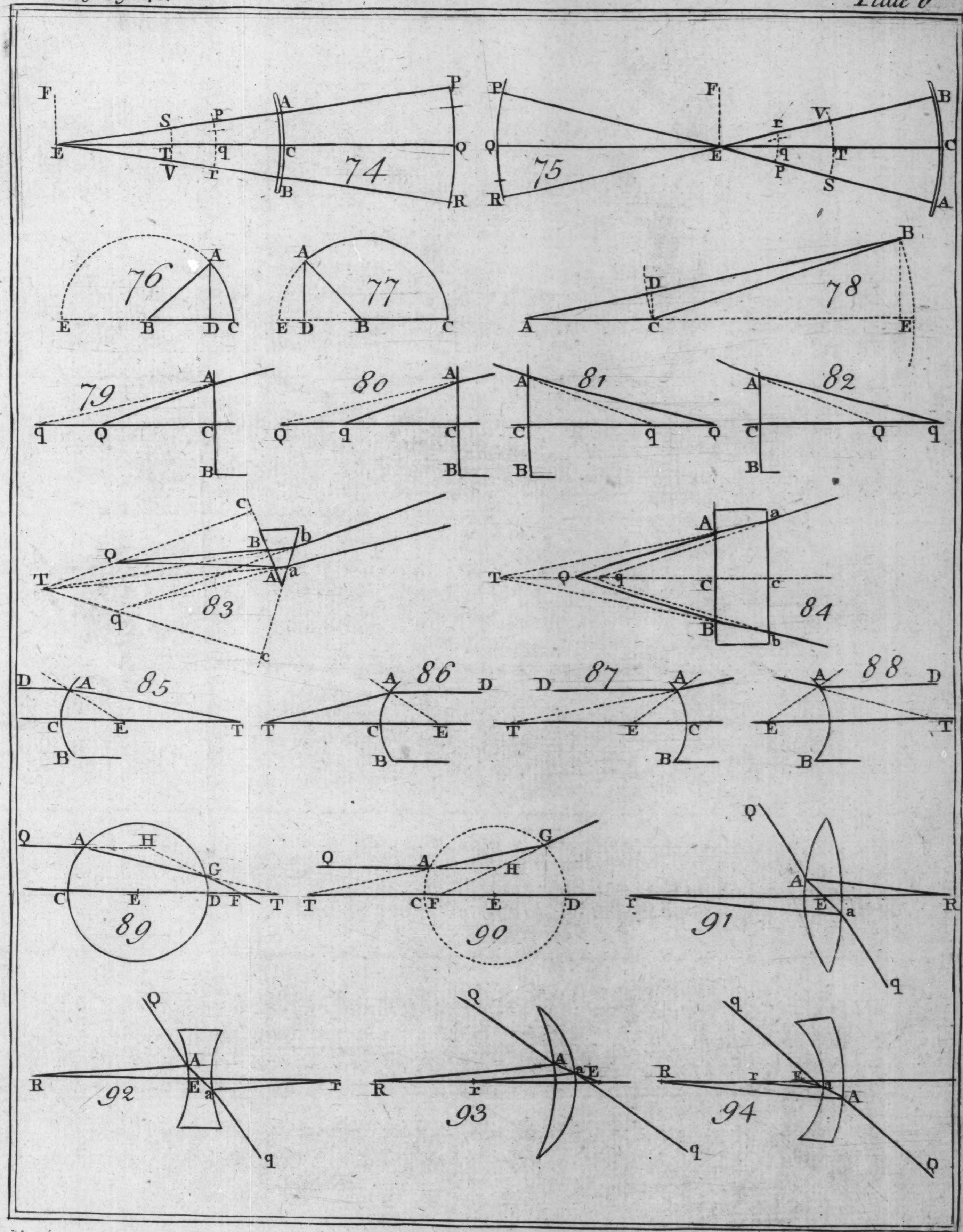
[c] Art. 92.

102. *Corol.* 4. Hence in particular in a double-convex or double-concave lens made of glass, it is as the sum of their semidiameters (or in a meniscus as their difference) to either of them, so is double the other, to the focal distance of the glass. For the continuations RT, rt are severally double their semidiameters: because in glass ET is to TR and also Et to tr as 3 to 2[d].

[d] Art. 93. 13.

103. *Corol.* 5. Hence if the semidiameters of the surfaces of the glass be equal, its focal distance is equal to one of them; and is equal to the focal distance of a plano-convex or plano-concave glass whose semidiameter is as short again. For considering the plane surface as having an infinite semidiameter, the first ratio of the last mentioned proportion may be reckoned a ratio of equality.

PRO-


74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94

PROPOSITION V.

104. *The focus of incident rays upon a ſingle ſurface, ſphere or lens being given, it is required to find the focus of the emergent rays.* Fig. 101. to 106.

Let any point Q be the focus of incident rays upon a ſpherical ſurface, lens, or ſphere, whoſe center is E; and let other rays come parallel to the line QEq the contrary way to the given rays, and after refraction let them belong to a focus F; then taking Ef equal to EF in the lens or ſphere, but equal to CF in the ſingle ſurface, ſay as QF to FE ſo Ef to fq, and placing fq the contrary way from f to that of FQ from F, the point q will be the focus of the refracted rays, without ſenſible error; provided the point Q be not ſo remote from the axis, nor the ſurfaces ſo broad as to cauſe any of the rays to fall too obliquely upon them.

For with the center E and ſemidiameters EF and Ef deſcribe two arches FG, fg cutting any ray $QAaq$ in G and g, and draw EG and Eg. Then ſuppoſing G to be a focus of incident rays, (as GA,) the emergent rays (as agq) will be parallel to GE [a]; and on the other hand ſuppoſing g another focus of incident rays (as ga,) the emergent rays (as AGQ,) will be parallel to gE. Therefore the triangles QGE, Egq are equiangular, and conſequently QG is to GE as Eg to gq; that is, when the ray $QAaq$ is the neareſt to QEq, QF is to FE as Ef to fq [b]. Now when Q accedes to F and coincides with it, the emergent rays become parallel, that is q recedes to an infinite diſtance; and conſequently when Q paſſes to the other ſide of F, the focus q will alſo paſs through an infinite ſpace from one ſide of f to the other ſide of it. *Q. E. D.*

105. *Corol.* 1. In refractions at a ſpherical ſurface AC, the focus q may alſo be found by this rule, as QF to FC ſo Cf to fq; becauſe FC and Ef and alſo FE and Cf are equal [c].

106. *Corol.* 2. It may alſo be found by this rule, as QF to QE ſo QC to Qq; placing Qq ſo that all the four diſtances from Q may lye on one ſide ſide of it, or elſe two on each. For the triangles QGE, QAq being equiangular we have QG to QE as QA to Qq.

107. *Corol.* 3. In a ſphere or lens the focus q may be found by this rule, as QF to QE ſo QE to Qq, to be placed the ſame way from Q as QF lyes from Q. For let the incident and emergent ray QA, qa be produced till they meet in e; and the triangles QGE, Qeq being equiangular, we have QG to QE as Qe to Qq; and when the angles of theſe triangles are vaniſhing, the point e will coincide with E; becauſe in the ſphere the triangle Aea is equiangular at the baſe Aa, and conſequently Ae and ae will at laſt become

[a] Art. 94. 99. 101.

[b] Art. 68.

[c] Art. 93.

come ſemidiameters of the ſphere. In a lens the thickneſs Aa is inconſiderable.

108. *Corol.* 4. In all caſes the diſtance fq varies reciprocally as FQ does; and they lye contrary ways from f and F; becauſe the rectangle or the ſquare under EF and Ef, the middle terms in the foregoing proportions, is invariable. Hence if either of the correſponding focuſes Q, q be put in motion along the axis of the pencil, the other focus will move the ſame way: and therefore if theſe focuſes be on contrary ſides of the glaſs, while one moves towards it the other will move from it; but if they be both on the ſame ſide of the glaſs, they will both move from it or both towards it; and will come nearer to each other as they come nearer to the glaſs, till when one coincides with its ſurface the other will do ſo too, in the ſingle ſurface accurately [a], and in the lens very nearly, provided the glaſs be very thin, and the diſtance of the ray from its axis be very ſmall [b]. But theſe focuſes cannot coincide at the ſurface of a globe; for QF and QE being finite, Qq cannot vaniſh [c]. They will coincide at the center of the ſingle ſurface, becauſe the rays fall perpendicularly, and therefore paſs through without ſuffering any refraction [d].

[a] Art. 105.
[b] Art. 104.
[c] Art. 107.
[d] Art. 15.

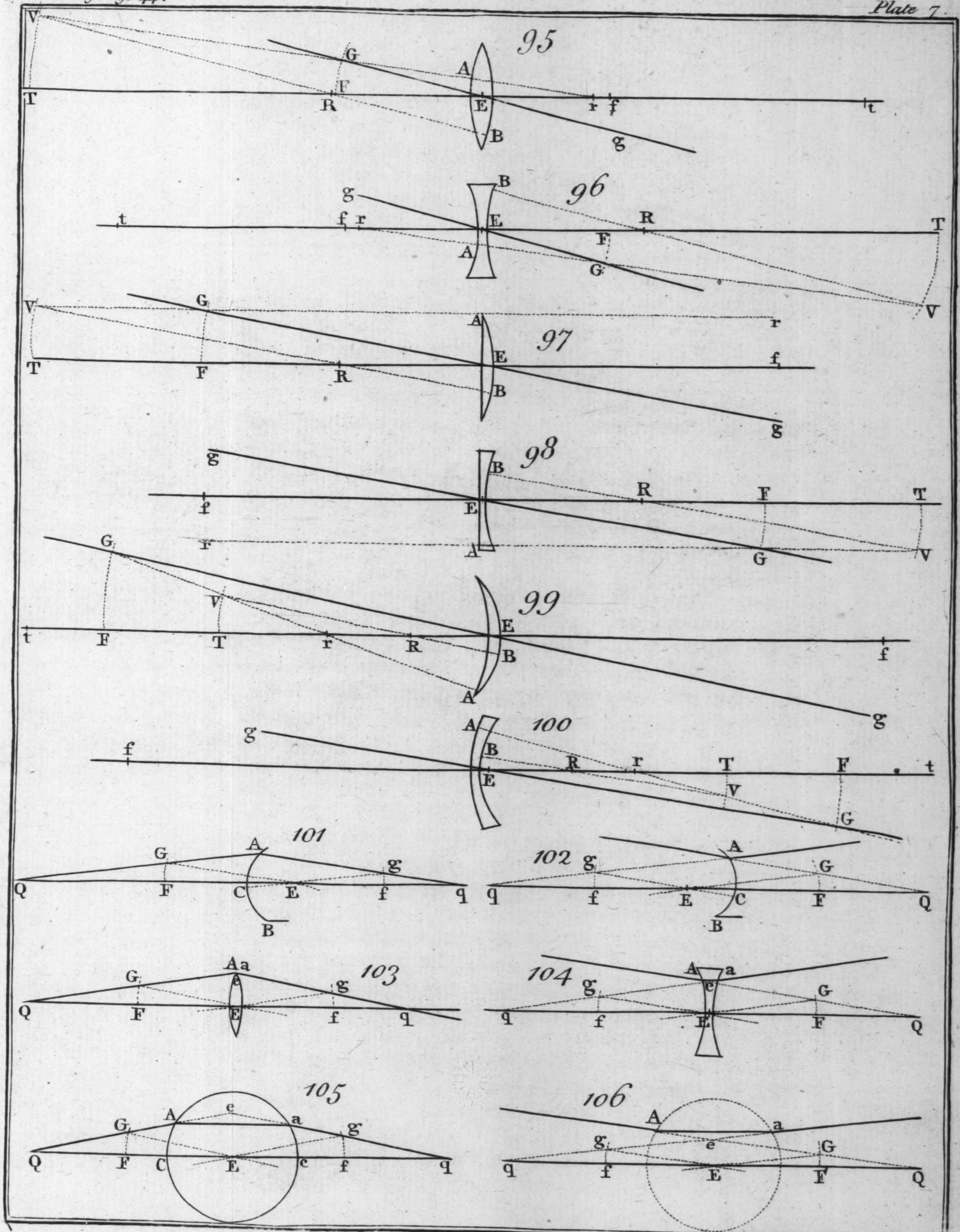
109. *Corol.* 5. Convex lenſes of different ſhapes that have equal focal diſtances, when put into each others places, have equal powers upon any pencil of rays to refract them to the ſame focus. Becauſe the rules abovementioned depend only upon the focal diſtance of the lens, and not upon the proportion of the ſemidiameters of its ſurfaces.

110. *Corol.* 6. The rule that was given for a ſphere of an uniform denſity, will ſerve alſo for finding the focus of a pencil of rays refracted through any number of concentrick ſurfaces, which intercede uniform mediums of any different denſities. For when rays come parallel to any line drawn through the common center of theſe mediums, and are refracted through them all, the diſtance of their focus from that center is invariable, as in an uniform ſphere.

111. *Corol.* 7. When the focuſes Q, q lye on the ſame ſide of the refracting ſurfaces, if the incident rays flow from Q, the refracted rays will alſo flow from q; and if the incident rays flow towards Q, the refracted will alſo flow towards q: and the contrary will happen when Q and q are on contrary ſides of the refracting ſurfaces. Becauſe the rays are continually going forwards.

The 75th and 76th articles are applicable to refractions as well as reflections.

" After


95
96
97
98
99
100
101
102
103
104
105
106

" After Des Cartes had discovered the true law of refraction[a], it " presently suggested to him the impossibility of collecting all the " rays of the same pencil into a single point by means of spherical " surfaces only. And not being aware that the aberration of rays " from the geometrical foci of the optical glasses then in use was " owing to any other cause than the *sphericalness* of their figures, he " immediately applied himself to investigate what curves could " make all the rays belonging to the same pencil converge after re- " fraction to a single point. The seeming importance of this pro- " blem induced other mathematicians after him to prosecute the " solution of it: and among other surfaces, it was found that those, " which are generated by the revolution of *conic sections* round their " axes, would produce the effect. The limits of an abstract will " not permit us to relate all their discoveries upon this subject; in " the following problems and their corollaries are contained the " principal discoveries of Des Cartes and his demonstrations of the " more difficult cases. *

[a] Art. 13.

"PROBLEM I.

" *To collect every ray belonging to a pencil of parallel rays into one " point, when they are incident upon the refracting surface of a denser " medium.*

" Take DK to HI as the sine of incidence to the sine of refrac- " tion; with the axis major DK and foci $H, I,$ describe an ellipse " DBK; describe also a spheroid by the revolution of this ellipse " round its transverse axis: and all the rays, which are incident " upon the convex surface DBb, parallel to DK, will converge to " the farther focus I. Fig. 107.

"PROBLEM II.

" *To collect every ray belonging to a pencil of parallel rays into one " point, when they are incident upon the refracting surface of a rarer " medium.*

" Take DK to HI as the sine of incidence to the sine of refrac- " tion; with the transverse axis DK and foci $H, I,$ describe an hyper- " bola; describe also an hyperboloid by the revolution of this hy- " perbola round its transverse axis: and all the rays, which are inci- " dent upon the concave surface BDY, parallel to DK, will con- " verge to the farther focus I. Fig. 108.

" DE-

* Des Cartes's Dioptr. Ch. 8.

Fig. 107, 108. " DEMONSTRATION. Let AB, MD be two parallel rays, of " which MD is perpendicular to the surface: join HB, IB; through " B draw two straight lines CBE, LBG, which are at right angles " to each other, and one of which CBE is a tangent to the curve " at the point of incidence B: through H draw HO parallel to LG, " meeting IB in the hyperbola, or IB produced in the ellipse, in O, " and the tangent in C. And the angles OBC, CBH being " equal [a], and HO cutting BC at right angles, the triangles OBC, " CBH are equal; and therefore OI is equal to the sum or difference of HB and BI; that is, OI equals DK [b]. Now LG " being perpendicular to the surface, the angle ABN is either the " angle of incidence or its complement to two right angles; and " therefore the sine of the angle BNI, which is alternate to ABN, " is equal to the sine of incidence [c]. And the sine of BNI is " to the sine of NBI as BI to NI [d]; that is, as OI to HI [e], or as " DK to HI; that is, as the sine of incidence to the sine of " refraction [f]. Wherefore the sine of NBI must be equal to " the sine of refraction, and BI is the course of the refracted ray. " *Q. E. D.*

[a] Hamil. Con. Sect. B. II. p. 16, 17.
[b] Hamil. Con. Sect. B. II. p. 14.
[c] Art. 83.
[d] Art. 85.
[e] Euc. VI. 2.
[f] By construction.

Fig. 109. " *Corol.* 1. If, therefore, with the center I and any radius IB, less " than ID, a circular arch BQ be described, and the figure BQD " revolve round its axis DQ, it will determine the shape of a piece " of glass, which, placed in air, will collect into one point I all the " rays that fall upon its convex surface parallel to DI. For the first " surface BDB of this solid refracts them all to I; and the radii of " a circle being perpendicular to its circumference, the course of " these rays will not be changed by the second surface BQB.

Fig. 110. " Likewise, if from any point B in the hyperbola the right line BQ " be drawn perpendicular to the axis, the figure BDQ by revolving " round DQ will generate a solid piece of glass, which, placed in " air, will have the same effect upon all rays that are incident upon " its plane surface parallel to the transverse axis.

" *Corol.* 2. Each of the glasses described in the preceding corollary " will make all the rays, which are incident diverging from the same " point, move parallel after refraction. For, if I be made the focus " of incident rays, the refracted rays will be parallel to ID by article. 11.

Fig. 111. " *Corol.* 3. Let an ellipse be constructed as before; and draw the " circular arch RO with the center I and any radius RI greater than " DI;

" *DI*; aſſume alſo any point *B* in the ellipſe, which is not farther " from *D* than from *K*, draw the right line *OB* tending to *I*; and " the glaſs, which is generated by the revolution of the figure " *BDRO* about its axis *DR*, will make all rays, that are incident " upon the concave ſide parallel to its axis, diverge from the point " *I* after refraction. For the mediums continuing the ſame, it is " evident that the ray *PB* is as much bent out of its courſe by the " concave ſide of the refracter *DB*, as the ray *AB* is by the convex " ſide of the ſame refracter [a]. Since therefore *AB* and *PB* are in [a] Art. 11. 17. " the ſame ſtraight line, and *AB* moves in the line *BI* after refrac- " tion, *BO*, which is in the ſame direction with *BI*, muſt be the " courſe of the ray *PB* after refraction.

" Parallel rays may alſo be refracted diverging accurately from " the ſame point by means of a glaſs conſtructed in the following " manner. Deſcribe an hyperbola *BDB* ſimilar to that which is Fig. 112. " deſcribed in the ſecond problem; aſſume any point *O*, from which " two ſtraight lines can be drawn in ſuch a manner that one of them " *OB* ſhall be parallel to the axis and cut the curve in ſome point *B*, " and that the other *OR* ſhall be perpendicular to the axis and not " cut the curve. Let the figure *OBDR* be turned round its axis, " and it will generate the ſolid required.

" *Corol.* 4. Each of the glaſſes conſtructed in the preceding corol- " lary will make all rays, that are incident converging to the ſame " point, move parallel after refraction; for the reaſon given in " corol. 2d.

" *Corol.* 5. If two glaſſes be conſtructed, each of which is ſimilar Fig. 113. " to the elliptic refracter deſcribed in the firſt corollary, and be placed " in air, ſo that their axes are in the ſame ſtraight line, and that their " convex ſurfaces are turned towards each other, all the rays, which " are incident diverging from the ſame point *I*, will after refraction " converge to the ſame point *i*.

" Or two hyperbolic refracters, each of which is ſimilar to the Fig. 114. " glaſs deſcribed in the firſt corollary, and placed in ſuch a manner " that their axes are parallel, and that their plane ſurfaces are turned " towards each other, will produce the ſame effect. And ſince the Fig. 115. " refractions at the curved ſurfaces will be the ſame, when the plane " ſurfaces coincide as when they are aſunder, a ſingle convex glaſs, " which is formed by turning the hyperbolic arches *DB*, *db* round " their axes, will collect into one point *i* all the rays which diverge " from *I*.

" *Corol.*

Fig. 116, 117. " *Corol.* 6. If it be required to make a pencil of rays, which are " incident diverging from a ſingle point *I*, diverge after refraction " from a different point *i*, uſe two elliptic glaſſes, the figures and " ſituations of which are repreſented in fig. 116, or the ſingle hy- " perbolic refracter, which is repreſented in fig. 117. The ſame " glaſſes will alſo make converging rays converge to a different point " after refraction.

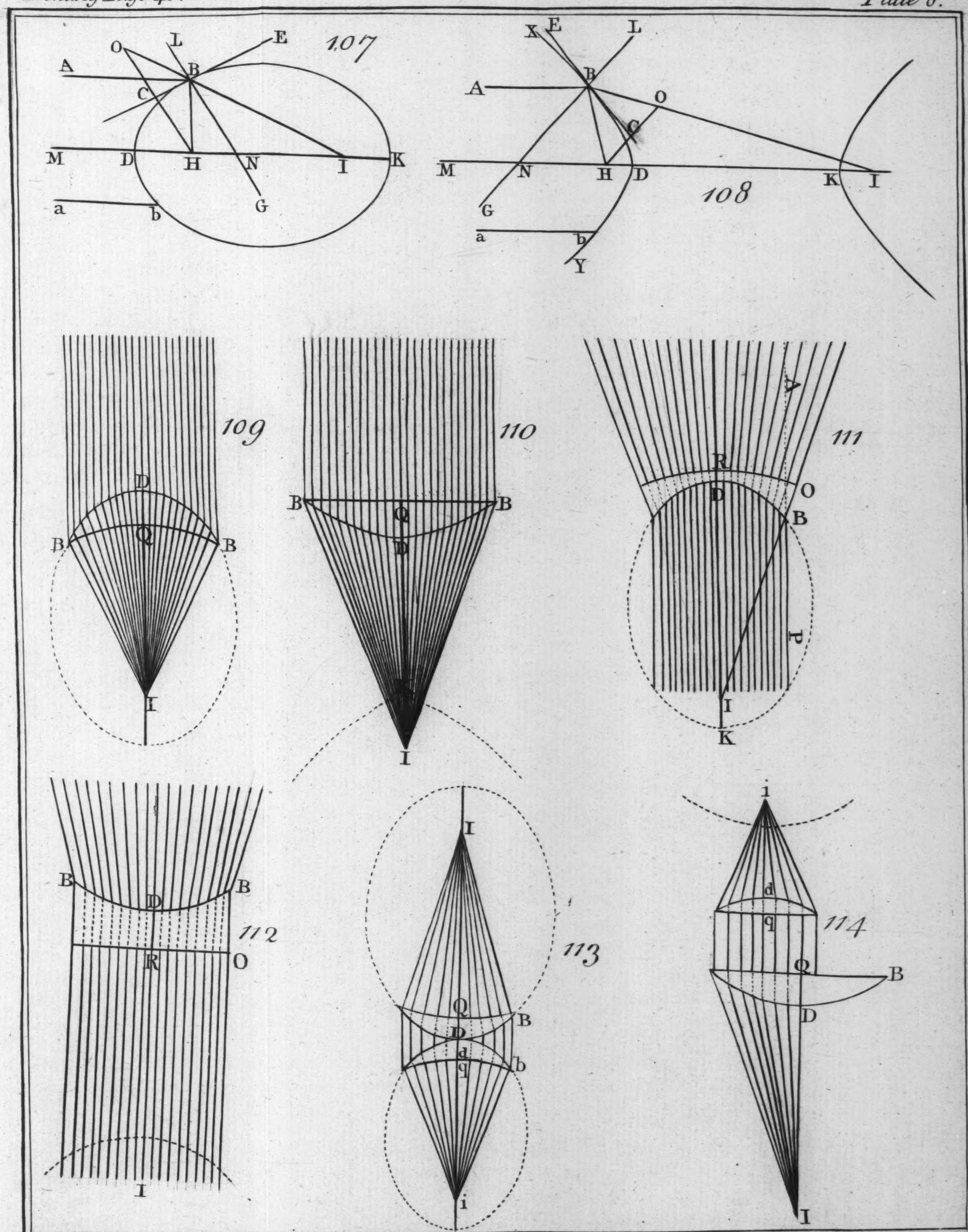
Fig. 118, 119. " *Corol.* 7. Laſtly, a pencil of converging rays will be made to " diverge accurately from a ſingle point after refraction, either by " means of two elliptic glaſſes, which are repreſented in fig. 118, or " by the ſingle hyperbolic glaſs repreſented in fig. 119.

" This apparent ſuperiority of conical ſurfaces induced not only " the mechanicks but the moſt eminent mathematicians to contrive " engines for grinding and poliſhing theſe ſurfaces; till Sir Iſaac " Newton diſcovered the different refrangibility of the rays of light [a]. " He then ſaw that, however accurately rays of the ſame colour " might be collected into a ſingle point, rays of different colours, " though they belong to the ſame incident beam, muſt move towards " different focuſes after refraction; and proved that the aberration " ariſing from the unequal refrangibility of different kinds of light " was 5449 times greater than the aberration, which is cauſed by the " *ſpherical* figure of a glaſs [b]. He obſerved beſides, that, though a " conical lens could refract ſuch rays as are parallel to its axis more " nearly to a ſingle point than a ſpherical ſurface, yet the latter " would be preferable to the former in reſpect of rays, which fall " with ſome degree of obliquity; ſince the curvature of a ſphere is " uniform in every part, whereas different parts of an ellipſe or hy- " perbola have a different curvature. For theſe reaſons and on ac- " count of the inſuperable difficulty that attended the conſtruction " of conical ſurfaces, all further attempts to correct the aberrations " in refractions by means of ſuch ſurfaces have long been diſcon- " tinued."

[a] Art. 28.

[b] Art. 220.

CHAP.


107
108
109
110
111
112
113
114

CHAP. VIII.

TO DETERMINE THE PLACE, MAGNITUDE AND SITUATION OF IMAGES FORMED BY REFRACTED RAYS.

PROPOSITION I.

112. IMAGES *formed by refractions at plane surfaces are similar to the objects, and are always erect, or in a similar situation to the objects, and on the same side of the planes *.*

Let *PQR* be an object radiating upon a refracting plane *ACB*; Fig. 120, 121. to which draw the perpendiculars *PA*, *QC*, *RB*, &c. and in these perpendiculars take *Ap* to *AP*, *Cq* to *CQ*, *Br* to *BR* in the given ratio of the sine of incidence to the sine of refraction[a]; and the focuses *p*, *q*, *r*, &c. will constitute a similar image in a similar situation to the object; the parts *pq*, *qr* being in the same ratio as *PQ*, *QR*. This is self-evident when the object is parallel to the refracting plane; and when it is inclined, produce it till it cuts the plane in *D*; and the produced image will also cut it in the same point *D*. For supposing the perpendicular *BrR* to move towards *D*, the lines *BR*, *Br* being in a given ratio will vanish both together: and because the triangle *pDP* is cut by parallel lines *qQ*, *rR*, it will be as *pq* to *PQ* (so *qD* to *QD*) so *qr* to *QR*[b]; and alternately *pq* to *qr* as *PQ* to *QR*. In like manner if the rays that belong to the focuses *p*, *q*, *r* be refracted again by another plane, either parallel or inclined to *AB*, their second focuses will constitute a second image, similar to the first image, and consequently similar to the object; and so on. *Q. E. D.*

[a] Art. 87.
[b] Euc. VI. 2.

PROPOSITION II.

113. *An image* pqr, *formed by a flat piece of glass* ABba *is upright, parallel and equal to the object* PQR, *and lies on the same side of the glass as the object; but nearer to it by a third part of the thickness of the glass.*

Let *PA*, *QC*, *RB* be drawn perpendiculars to the surface *ACB*, Fig. 122, 123. and the focuses *p*, *q*, *r* of the several pencils that flow from *P*, *Q*, *R*,

* This proposition is only true, when the object is a straight line or a plane superficies.

[a] Art. 87. lying in these perpendiculars[a], the image must be upright. But we have also shewn, that each of the focuses p, q, r, lyes nearer to ACB, than the points P, Q, R, by a third part of Cc[b]; therefore the image must be so much nearer than the object, and parallel to it. *Q. E. D.*

[b] Art. 91.

PROPOSITION III.

114. *An image formed by a prism is always upright, and equal to the object, and lies on the same side of the prism, and at the same distance from it as the object itself; provided the refracting angle of the prism, and the refractions made by it, be but small.*

Fig. 124. Take two rays, PE, QE, which coming from the extremities of the object, pass through a point E, so near to the angular point of the refracting angle, that the distances between their points of incidence and emergence need not be mentioned. And since the total bendings of the rays PEN, QEO are equal[c], they will cross each other, so as that the angle PEQ will be equal to the angle NEO or to pEq, made by the emergent rays produced backwards: and because the distance Ep of the focus p, of the pencil that flowed from P, is equal to EP[d], and in like manner the focal distance Eq equal to EQ; the image pq will be upright and equal to the object, and at an equal distance on the same side of the prism. *Q. E. D.*

[c] Art. 55.

[d] Art. 90.

PROPOSITION IV.

Fig. 125. to 128. 115. *If an arch of a circle* PQR *described upon the center* E, *of a spherical surface, sphere or lens, be considered as an object, its image* pqr *will be a similar concentrick arch; whose length will be to the length of the object in the ratio of their distances from the common center* E; *and the image will be erect or inverted, with respect to the object, according as they lye on the same side of the center or on contrary sides.*

The proposition is evident by inspection of the 125th figure in all cases of refractions made by concentrick surfaces; because the parts of these surfaces are alike exposed to the parts of the concentrick object. And in a lens the focuses of all the pencils of parallel rays lye also in a concentrick arch GFH; whence Pp and Qq being third proportionals to two pair of equal distances PG and PE, QF and QE[e], are also equal; and so the image pqr is also a concentrick arch. Now since the axes of the pencils are reckoned straight lines passing through E[f], the angles pEr, PER are equal; and therefore the ratio of the image to the object, is the same as of their distances from

[e] Art. 107.

[f] Art. 97.

from the center *E*. And according as their correſponding extremities *P*, *p* are on the ſame or on contrary ſides of *E*, they lye on the ſame or contrary ſides of their middle points *Q*, *q*; that is, the image is accordingly erect or inverted. *Q. E. D.*

116. *Corol.* The ſmaller the circular object is with reſpect to its radius or diſtance from the center *E*, the nearer it approaches in ſhape to a ſtraight line, and ſo does its ſimilar image. Conſequently a ſmall ſtraight object placed at a good diſtance from the center of the glaſs may be reckoned to have a ſtraight image very nearly[a]: though in ſtrictneſs of geometry it is an arch of a conic ſection. And by theſe propoſitions the images of all objects may be determined, by finding the images of their out-lines. [a] Art. 68.

CHAP. IX.

CONCERNING THE EYE AND MANNER OF VISION.

117. CONSIDERING what has been ſaid in the 92d and 115th articles, one might contrive a tolerable eye in this manner, by placing a pellucid hemiſphere *ABC* to ſerve for the fore part, and another concentrick one *DqE*, oppoſite to the former, to ſerve for its bottom or back part; making the ſemidiameter, *Oq*, of the latter triple the ſemidiameter, *OB*, of the former; and then by filling the whole cavity of both with water. By this means rays of light flowing from the points *P*, *Q*, *R*, &c, of remote objects, after refraction at the ſurface *ABC*, will be collected to as many other points *p*, *q*, *r*, of the cavity *DqE*, and paint an image upon it. And becauſe a ſpherical ſurface does not accurately refract all the rays of a large pencil to a ſingle point[a], but only thoſe that go pretty near its axis; this imperfection might be remedied by covering the baſe *AC*, of the leſſer hemiſphere, all but a moderate hole about the center *O*; which would anſwer the purpoſe much better than if the ſurface itſelf was covered, all but a hole in the middle about *B*. For in this latter caſe the ſurface *ABC* would not receive rays from the lateral points *P*, *R*, ſo directly as thoſe from the middle of the object, to all which it is expoſed alike when the hole is left open at the center *O*.

A fictitious eye deſcribed by *Hugens* *. Fig. 129.

[a] See the latter part of chap. 7.

118. Though this conſtruction of the eye appears not amiſs at firſt ſight, yet we ſhall ſee preſently that the author of nature has

And compared to the natural eye.

* Opuſcula Poſthuma. p. 112.

 wiſely

wisely varied some things for the better, and added others absolutely necessary; though in every thing we cannot perceive his designs. In the first place he would not make use of an entire hemisphere *ABC*, but retaining the middle part, has taken off pretty much from the sides, and yet without contracting the compass of objects taken in at one view. The reason of this was to bend inwards the edges of the larger hemisphere about *D* and *E*, thereby reducing the shape of the eye to a rounder figure, for the convenience of its motion every way in the cavity that contains it. He has therefore given it such a shape, as is expressed in this other figure, representing an human eye dissected through its axis, all the parts being twice as big as in the life to render them more conspicuous.

Fig. 130.

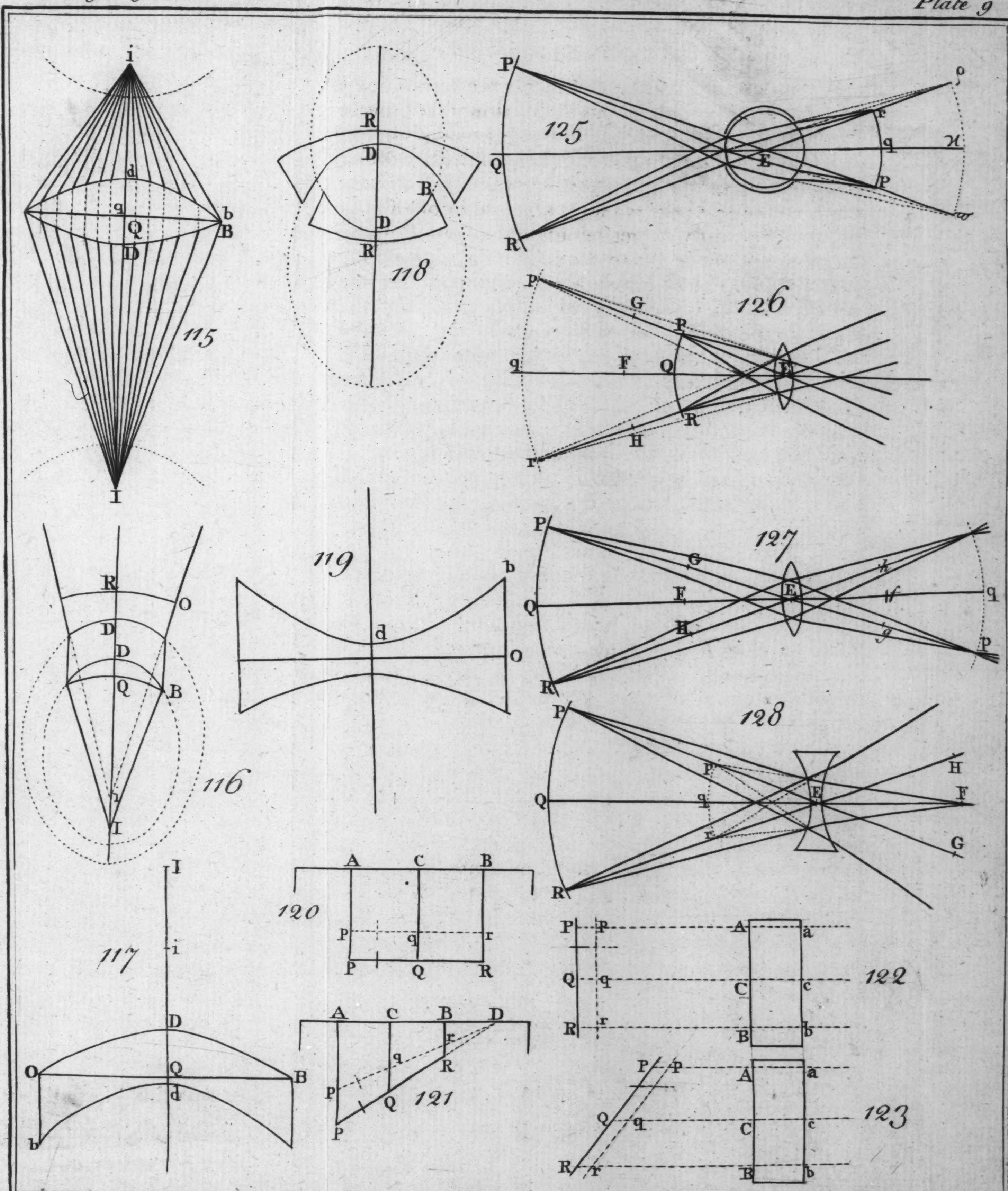
An human eye described.

119. Here the transparent parts of the coat called the cornea is *ABC*; the remainder *ATYC* being opake, and a portion of a larger sphere. Within this outward coat anatomists distinguish two others; the innermost of which is called the retina, being like a fine net composed of the fibres of the optick nerve *YVT* woven together, and is white about the parts *p*, *q*, *r*, at the bottom of the eye. The cavity of the eye is not filled with one liquor, but with three of different sorts. That contained in the outward space *ABCOEGFDO* is called the aqueous humor, being perfectly fluid like water; the other contained in the inward space *EpqrDFG* is a little thicker like the white of an egg, and is called the vitreous humor; the third humor *FG* is shaped like a lens of unequal convexities, lying between the two former, and fixed to the side coats by filaments or threads extended all round it, and is called the crystalline humor, being hard like the white of an egg boiled, but as clear as the other two, and differs from them in a greater degree of refractive power; whereby the rays that came from the points *P*, *Q*, *R*, having received a degree of convergence by the refraction of the cornea *ABC*, are made to converge a little more by other refractions at the surfaces of the crystalline *FG*; so that uniting in as many other points *p*, *q*, *r*, upon the retina, they represent the points of the object *P*, *Q*, *R*, from whence they came. And perhaps the rays are so directed by these secondary refractions at the crystalline, as to fit the cavity *pqr* intended to receive them; which otherwise must have been a portion of a larger sphere [a], according to the fictitious design in the former figure.

[a] Art. 111. latter part.

The crystalline makes all pictures distinct.

120. Besides this there was greater need of the lens *FG* upon another account; namely, to help the eye to conform itself for seeing objects distinctly at all distances, which was wanting in the fictitious eye. There are two ways of doing it by the help of this lens *FG*, in order


115
116
117
118
119
120
121
122
123
125
126
127
128

order to ſee things near at hand; either by moving it nearer to the outward cornea, or by increaſing its convexity, or perhaps by doing both at once. If it is moved towards the cornea, this may be effected by the preſſure of the muſcles againſt the ſides of the eye, and conſequently againſt the vitreous humor; but if the cryſtalline alters its figure and becomes rounder for ſeeing near objects, the filaments *DF, EG*, whoſe greater tenſion helps to flatten it, may perhaps be ſlackened by the lateral preſſure aforeſaid; and poſſibly both theſe alterations are made at the ſame time. The hole or pupil *O* is not placed in the center of the cornea *ABC*, as in the fictitious eye, but ſomewhat nearer to its front. The reaſon is uncertain, unleſs this alſo may contribute to make the images coincide with the cavity of the retina, (in all their parts,) which otherwiſe muſt have been ſhaped according to a larger ſphere [a].

[a] Art. 111. latter part.

Some dimenſions of an human eye.

121. The diameter *AY* of the ſphere of the eye is about an inch of the Rheinland foot, which is much the ſame as the old Roman foot: and the diameter of the outward cornea is about three fifths of an inch; the breadth of the pupil, *O*, has no fixt meaſure, being greater or ſmaller, as any one may try, according as leſs or more light ſhines upon the eye; it alſo contracts at the near approach of any ſmall object, when we endeavour to view it diſtinctly. Its fabrick is admirable in this reſpect that while it alters its ſize it preſerves its roundneſs [1]. So far Mr. *Hugens* [2], to which I add ſomething from Sir *Iſaac Newton* [3].

Pictures on the retina the cauſe of viſion.

122. This account of the eye and of the cauſe of viſion is farther confirmed by theſe arguments; that anatomiſts when they have taken off from the bottom of the eye that outward and thickeſt coat called the dura mater, can ſee through the thinner coats the pictures of objects lively painted thereon. And theſe pictures propagated by motion along the fibres of the optick nerves into the brain, are the cauſe of viſion. For according as theſe pictures are perfect or imperfect, the object is ſeen perfectly or imperfectly. If the eye be tinged with any colour (as in the diſeaſe of the jaundice) ſo as to tinge the pictures in the bottom of the eye with that colour, then all objects appear tinged with the ſame colour.

Confuſed pictures in old men's eyes how cauſed, and mended by convex glaſſes. Fig. 131.

123. If the humors of the eye decay by old age ſo as by ſhrinking to make the cornea and coat of the cryſtalline humor grow flatter than before, the light will not be refracted enough, and for want of a ſufficient refraction will not converge to the bottom of the eye,

1 See this accounted for in Mr. *Cheſſelden*'s Anatomy, p. 319. 3d Ed.

2 Dioptrica prop. 31.

3 Opticks pag. 12.

but

but to ſome place beyond it; and by conſequence will paint in the bottom of the eye a confuſed picture; and according to the indiſtinctneſs of the picture the object will appear confuſed. This is the reaſon of the decay of ſight in old men, and ſhews why their ſight is mended by ſpectacles. For the convex glaſſes ſupply the defect of plumpneſs in the eye, and by increaſing the refractions make the rays converge ſooner, ſo as to convene diſtinctly at the bottom of the eye, if the glaſs has a due degree of convexity.

Fig. 132.

Confuſed pictures in ſhort-ſighted eyes, how cauſed and mended by concave glaſſes. Fig. 133. [a] Art. 108. Fig. 134.

124. And the contrary happens in ſhort-ſighted men, whoſe eyes are too plump. For the refraction being now too great, the rays converge and convene in theſe eyes before they come at the bottom; and therefore the picture made in the bottom and the viſion cauſed thereby will not be diſtinct, unleſs the object be brought ſo near the eye as that the place where the converging rays convene may be removed to the bottom[a]; or that the plumpneſs of the eye be taken off and the refraction diminiſhed by a concave glaſs, of a due degree of concavity, as is repreſented in fig. 134; or laſtly that by age the eye grows flatter till it comes to a due figure. For ſhort-ſighted men ſee remote objects beſt in old age, and therefore they are accounted to have the moſt laſting eyes. So far Sir *Iſaac Newton*.

Glaſſes for defective eyes determined.

125. In order to determine the propereſt glaſſes for defective eyes, the limits of confuſed and diſtinct viſion, or the diſtances of thoſe places from the eye, where an object begins to appear confuſed, may be found by meaſuring the leaſt diſtance from which a long-ſighted perſon can ſee a pretty large print diſtinctly and read it readily: and likewiſe by meaſuring the greateſt and the leaſt diſtances, from which a ſhort-ſighted perſon can ſee a ſmall print diſtinctly and read it readily: or ſtill more exactly by placing the end of a long ruler cloſe to the eye, or rather a little under it, and by obſerving the greateſt and leaſt diſtances at which the lines drawn lengthways along the ruler, begin to appear confuſed. I ſhall call thoſe glaſſes the propereſt for defective eyes, which are the leaſt concave or the leaſt convex of any that will anſwer the purpoſe of diſtinct viſion; for a reaſon to be mentioned hereafter.

Fig. 135.

126. Let Eq be the leaſt diſtance from which any ſmall objects appear diſtinct to the eye of a long-ſighted perſon; and EQ the leaſt diſtance from which he wants to ſee them diſtinctly. Towards q take QF to QE as QE to Qq, and EF will be the focal diſtance of a convex lens, which being put cloſe to his eye, will make him ſee an object diſtinctly at any place between Q and F, and poſſibly beyond F. For the rays that flow from Q will emerge from the glaſs and will enter the eye as if they had come directly from q, to the naked eye;

eye[a]; and supposing Q to recede from the eye, q will also recede from it to infinity through places where the naked eye can see distinctly[b]. And therefore the refracted rays, diverging as from these places, will also produce distinct vision of the object Q as far as to F; and still farther if the person can see distinctly by converging rays.

[a] Art. 107.

[b] Art. 108.

127. Therefore if he wants to see distinctly from no less a distance than half Eq, that is, only as near again as with his naked eye, a convex lens whose focal distance is Eq will be the properest; and will make him see an object distinctly at any distance not less than half Eq. For supposing Qq and QE to be equal, the point F will fall upon q by the foregoing proportion.

Glasses for short sighted persons. Fig. 136.

128. Let EF be the greatest distance from which an object at F appears distinct to the eye of a short-sighted person; and it will also be the focal distance of a concave lens, which being put close to his eye at E, will be the properest for seeing remote objects distinctly. Because the rays of a pencil, which come from any remote object, and consequently fall parallel upon the lens, will emerge from it to the eye, as if they had come directly to the naked eye from an object at F. And consequently the picture of the remote object formed upon the retina by rays refracted through the lens, will be as distinct as the picture of an object at F seen by unrefracted rays.

Fig. 137.

129. Let Eq be the least distance from which the same person can see an object distinctly with his naked eye; then say as QF to QE so QE to Qq, and placing Qq towards F, the point q will be the nearest point, which he will be able to see distinctly through the lens abovementioned. For by art. 107, the rays of a pencil which fall upon the lens converging towards Q, will after refraction converge to q; and on the contrary, the rays which flow from q will emerge from the lens diverging from Q; and supposing the point q to recede from the eye, the point Q will also recede from it to such places where the naked eye can see distinctly[b]. But if the point q approaches towards the eye, the point Q will also approach towards it, to such places where the naked eye cannot see distinctly, by the supposition.

[b] Art. 108.

130. Consequently if QF, the space between the limits of confused vision, be not less than QE, that one glass whose focal distance is EF, will make all objects appear to him distinct which are any where placed beyond F, the reach of his naked eye. For in this case Qq cannot be greater than QF, as is manifest by the proportion above.

Fig. 138.

131. But if he wants a pair of concave spectacles to read or write with, let the distance Eq be no greater than what is convenient for that purpose, and let QF be the limits of confused vision as before; and

and towards q take FG to FE as FE to Fq, and a concave lens whoſe focal diſtance is EG will be the propereſt for this purpoſe. For by art. 107, the rays of a pencil which fall on this glaſs converging towards F will converge to q after refraction, and on the contrary; and therefore he will ſee an object diſtinctly as far off as q; and alſo nearer than F, if QF be but half of EF. For ſuppoſing rays to fall on the lens converging towards Q, ſay as QG to QE ſo QE to QH, and the refracted rays will converge to H and conſequently H will be the neareſt point that can be ſeen diſtinctly through this glaſs. But if Q biſects EF it is manifeſt that QH is leſs than QF; becauſe QG, QF, QH are now continual proportionals.

Directions for the choice of convex and concave ſpectacles.

132. Thus any perſon may be fitted with the propereſt glaſſes though he lives at a diſtance from the ſhops where they are ſold, by ſending the workman their focal diſtances computed by the foregoing rules. But if choice of glaſſes be at hand, he may be better fitted by trial; obſerving only to uſe thoſe glaſſes which are the leaſt concave or the leaſt convex of any that will fit the eye. Theſe are the glaſſes which I have computed and called the propereſt. For ſince they cannot be put quite cloſe to the eye, the leſs any glaſs is concave, the leſs it diminiſhes the pictures of objects upon the retina. It will alſo accuſtom the eye to that conformation of its coats and humors, which is proper for ſeeing objects as far off as it can; and conſequently may prevent the eye from growing more and more ſhort-ſighted. On the other hand, the leſs any glaſs is convex, the leſs it magnifies the pictures of objects upon the retina; and alſo obliges the eye to that conformation, which is requiſite for ſeeing objects as near as it can. Both which may prevent the eye in ſome meaſure from growing more and more long-ſighted. For when the picture upon the retina is very large, it need not be quite ſo diſtinct, as when it is ſmaller, to convey an idea of the ſame number of parts of an object; and conſequently the eye will be more at liberty to recede from that conformation, which is proper for the glaſs; and to relapſe into that to which it inclines, and which is only proper for ſeeing remote objects.

Equal parts of a ſmall object ſubtend equal angles at the eye. Fig. 139.

133. When the perpendicular ſubtenſe BC of a ſmall angle BAC is divided into any number of equal parts BH, HI, IC, the lines, HA, IA, drawn from the points of diviſion to A, will divide the angle BAC into the ſame number of parts, which will be nearly equal among themſelves. For they would be ſo exactly if the line BC was an arch of a circle, drawn upon the center A; from which it differs ſo much the leſs as the angle at A is ſmaller; and ſo the propoſition is exacteſt in the ſmalleſt angles.

134. When

134. When the distance AB is double or treble of Ab, the subtense BC will be double or treble of the subtense bc of the same angle at A[a]. Divide BC into its parts BH, HI, IC, each equal to bc, and the rays HA, IA, will divide the angle BAC into as many equal parts[b]. Therefore when two angles bAc, BAH are subtended by the same or by equal lines bc, BH, the magnitude of the first angle bAc, will be to the magnitude of the second BAH, as the second distance BA to the first distance bA.

Small angles subtended by the same perpendicular are reciprocally as its distances from the angular points.

[a] Art. 24.
[b] Art. 133.

135. In determining the magnitude of pictures upon the retina, only one ray in each pencil need be considered; because when the picture is distinct, all the rays in any one pencil are collected to one and the same point of the retina. Or, which is much the same, we may suppose the pupil of the eye contracted to a point: and, for greater simplicity and ease of the imagination that this point O is a little hole at the center of a dark, hollow hemisphere DqE, admitting only single rays straight through it without any refraction at all. For then the lengths of these pictures pqr will increase and decrease as the angle pOr does, or as the angle POR does; which I am going to shew to be the property of the natural eye[c]: and if the semidiameter Oq, of this hollow hemisphere, be about $\frac{5}{8}$ of an inch, or of the axis of an human eye, the pictures of the same objects will always have the same bigness in both sorts of eyes[c], very nearly.

The pupil of the eye may be considered as a point.

Fig. 129.

[c] Art. 136.
[d] Art. 142.

136. The diameters or lengths of the pictures of objects upon the retina are measured by, or proportionable to, the angles which the rays that come from the extremities of the object do make in falling on the eye; provided those angles be but small. For let two or more objects $P\mathcal{Q}$ and $\varpi\varkappa$, either parallel or oblique to each other, subtend the same angle $PO\mathcal{Q}$ or $\varpi O\varkappa$ at O; and because the particles of light flowing from P and ϖ describe the same line $P\varpi O$, they will be refracted to the same point p upon the retina; and in like manner those that flow from $\mathcal{Q}$ and $\varkappa$ will be refracted to the same point q; and so the pictures pq of the objects $P\mathcal{Q}$, $\varpi\varkappa$, which subtend the same angle at O, are the same in magnitude; which was the first thing to be proved.

Diameters of pictures on the retina are as the angles subtended by the object at the eye.

Fig. 140.

Now the pictures of objects painted upon the retina of a dead eye are found by experience to be perfectly well shaped and proportioned in their parts[e]; that is, the proportion of the parts pq, qr, of the whole picture pqr, is the same as that of the parts $P\mathcal{Q}$, $\mathcal{Q}R$, of the whole object $P\mathcal{Q}R$, and this latter proportion is very nearly the same as that of the angles $PO\mathcal{Q}$, $\mathcal{Q}OR$ subtended by the parts $P\mathcal{Q}$, $\mathcal{Q}R$[f]; and so the proposition is proved when the objects $P\mathcal{Q}$, $\mathcal{Q}R$ are both at the same distance from the eye. And since it was shewn just before,

[e] Art. 122.
[f] Art. 133.

fore, that the objects PQ and $\varpi\varkappa$ have the same picture pq, it follows that the proportion of the pictures of the objects $\varpi\varkappa$ and QR is the same as that of the angles $\varpi O\varkappa$, QOR, subtended by them at the eye.

Are reciprocally as the distances of the object from the eye.

137. When an object approaches towards the eye, the diameter of its picture upon the retina increases in the same proportion as the distance between the eye and the object decreases; and on the contrary, it decreases in the same proportion as that distance increases. For the diameter of its picture increases in the same proportion as the angle increases, which the object subtends at the eye[a]; and this angle, when small, increases in the same proportion as the distance between the eye and object decreases[b].

[a] Art. 136.

[b] Art. 134.

Brightness of pictures not altered by the distance of the object from the eye.

138. The degree of brightness of the picture of an object painted upon the retina continues the same, at all distances between the eye and the object; provided none of the rays be stopt by the way, and that the pupil does not alter its aperture. For instance, when the eye approaches as near again to the object, the picture upon the retina becomes double in length and double in breadth, and consequently quadruple in surface; for the surface would be double, if its length alone or breadth alone was double. The quantity of rays received through the same aperture of the pupil, at half the distance from the object, is also quadruple[c]: and being equally spread over four times the quantity of surface of the retina, they are just as dense as before when the object was at twice the distance.

[c] Art. 25.

Faintness of pictures of remote objects how caused.

139. It follows then that the faint appearance of remote objects is owing to the opacity of the atmosphere, which hinders part of their light from coming to the eye. Accordingly we find that the sun, moon, and stars appear very faint when near the horizon, and brighter continually as they rise higher; because the tract of vapours, which lies in the way of the rays, is longest and thickest near the horizon; and becomes thinner and shorter as the objects rise higher, and consequently does less obstruct the passage of the rays.

Their degrees of brightness by day-light and moon-light compared.

140. The sensibility of the eye, or its power to discern objects, without inconvenience, by different quantities of light, is vastly extensive. For instance, the disproportion in the quantities of light, cast upon the horizon by the sun and moon, at any equal altitudes, I find is no less than 90 thousand to 1, when the moon is full; or no less than 180 thousand to 1, when the moon is in the quarters. And the proportion between those parts of the lights of the sun and moon, whatever they be, which are reflected to our eyes from the same object by day and by night, can hardly be different from the proportion of the whole lights. Allowing then that the aperture of the pupil

pupil may poſſibly be 8 or 9 times leſs by day than by night, (that is about 3 times leſs in diameter,) yet the proportion in the quantities of day-light and moon-light, received by the eye from the ſame object, to illuminate a picture of the ſame bigneſs, will be no leſs than 20 thouſand to 1, when the nights have a middle degree of moon-light; I ſay no leſs, becauſe the numbers here given are deduced from a rule, which is built upon this principle; that the moon reflects all the light received from the ſun; which cannot be true, by reaſon of the appearance of very large obſcure places in her body; and in all probability a great part of the incident light is buried and loſt even in the brighteſt places.

The rule I mentioned is this, day-light is to moon-light as the ſurface of an hemiſphere, whoſe center is at the eye, to the part of that ſurface which appears to be poſſeſſed by the enlightened part of the moon: ſo that the whole heavens covered with moons would only make day-light. This will be evident enough from the following conſiderations, though I invented it another way. Day-light is made by innumerable reflections of the ſun's rays from all ſorts of bodies till at laſt they come to our eyes: for if this were not ſo, we could ſee nothing in the world, even in the day time, but the ſun and ſtars and ſelf-ſhining ſubſtances [a]. Accordingly we find that day-light is much the ſame, whether the ſun ſhines out or not, in the place we are in; becauſe his light is reflected to us from a vaſt quantity of earth, air and clouds extended all round us, perhaps to a hundred miles or more. So that the abſence of the ſun's rays from a particular place ſcarce alters day-light. Another thing is that the moon by day appears like a cloud in the air of a middle degree of brightneſs; ſome appearing duller and ſome brighter than the moon itſelf. The rays of the ſun being therefore intercepted in the night from all the viſible clouds, and being reflected to us by the moon only, it follows that day-light is to moon-light, as the apparent ſurfaces of all the viſible clouds, to the apparent ſurface of the viſible part of the moon, conſidered as the only cloud which remains enlightened. And theſe two lights, whatever be the diſtances of the moon and clouds, are juſt the ſame as if thoſe bodies were all placed at any equal diſtances from us, and compoſed the ſurface of an hemiſphere [b]; whoſe parts are the true meaſures of the parts of the light which comes to us.

[a] Art. 2.

[b] Art. 138.

141. A vaſt diſproportion between the lights of the ſun and moon appears alſo by experiments made with burning-glaſſes; either by refraction of the rays through very broad lenſes, or by reflection from very broad concave-glaſſes or metals: which by collecting the rays

And confirmed by experiments with burning-glaſſes.

rays of the ſun into a ſmall round image at the focus, do excite a more violent heat and burn quicker than the hotteſt wind-furnaces: as appears by their melting and calcining the hardeſt metals, and by vitrifying bricks and ſtones, in much leſs portions of time than a minute [1]. Yet the rays of the moon being collected by the ſame glaſſes, do not excite the leaſt ſenſible heat; nor do they ſenſibly affect the niceſt thermometer, when caſt upon the ball of it [2], though the brightneſs of the light be very ſenſibly increaſed. By meaſuring the breadth of the round image at the focus, and by comparing it with the breadth of the glaſs itſelf, it appears that ſome of theſe burning-glaſſes collect the incident rays into a ſpace about 2 thouſand times leſs than they poſſeſſed at their incidence. But by the preceding calculation, the light of the full moon muſt be condenſed about 90 thouſand times [a], to make it as denſe and as warm as the direct rays of the ſun. It is no wonder then that the heat of the moon's rays is not ſenſible in the focus of the glaſs, being then even 40 or 50 times thinner than the direct rays of the ſun. For it is found by experiments made with theſe glaſſes that the degrees of heat are proportionable to the denſities of the rays: which being compared with a ſcale of the degrees of heat and warmth of ſeveral natural bodies, determined by Sir *Iſaac Newton*, in the philoſophical tranſactions [3], it appears there is a vaſt diſproportion between the degrees of light which the eye can bear and be ſenſible of, and the degrees of its heat which the touch can bear and be ſenſible of.

a Art. 140.

Viſion limited both by magnitude and diſtance.

142. Dr. *Hook* aſſures us that the ſharpeſt eye cannot well diſtinguiſh any diſtance in the heavens, ſuppoſe a ſpot of the moon's body, or the diſtance of two ſtars, which ſubtends a leſs angle at the eye than half a minute; and that hardly one of a hundred can diſtinguiſh it when it ſubtends a minute [4]. If the angle be not bigger than this, the two ſtars appear to the naked eye, as if they were but one. I have been preſent at making the experiment, when a friend of mine, who had the beſt eyes of all the company, could ſcarce perceive a white circle upon a black ground, or a black circle upon a white ground, or againſt the ſky-light, when it ſubtended a leſs angle at the eye than two thirds of a minute; or which is the ſame thing, when its diſtance from the eye exceeded 5156 times its own diameter: which agrees well enough with Dr. *Hook*'s obſervation.

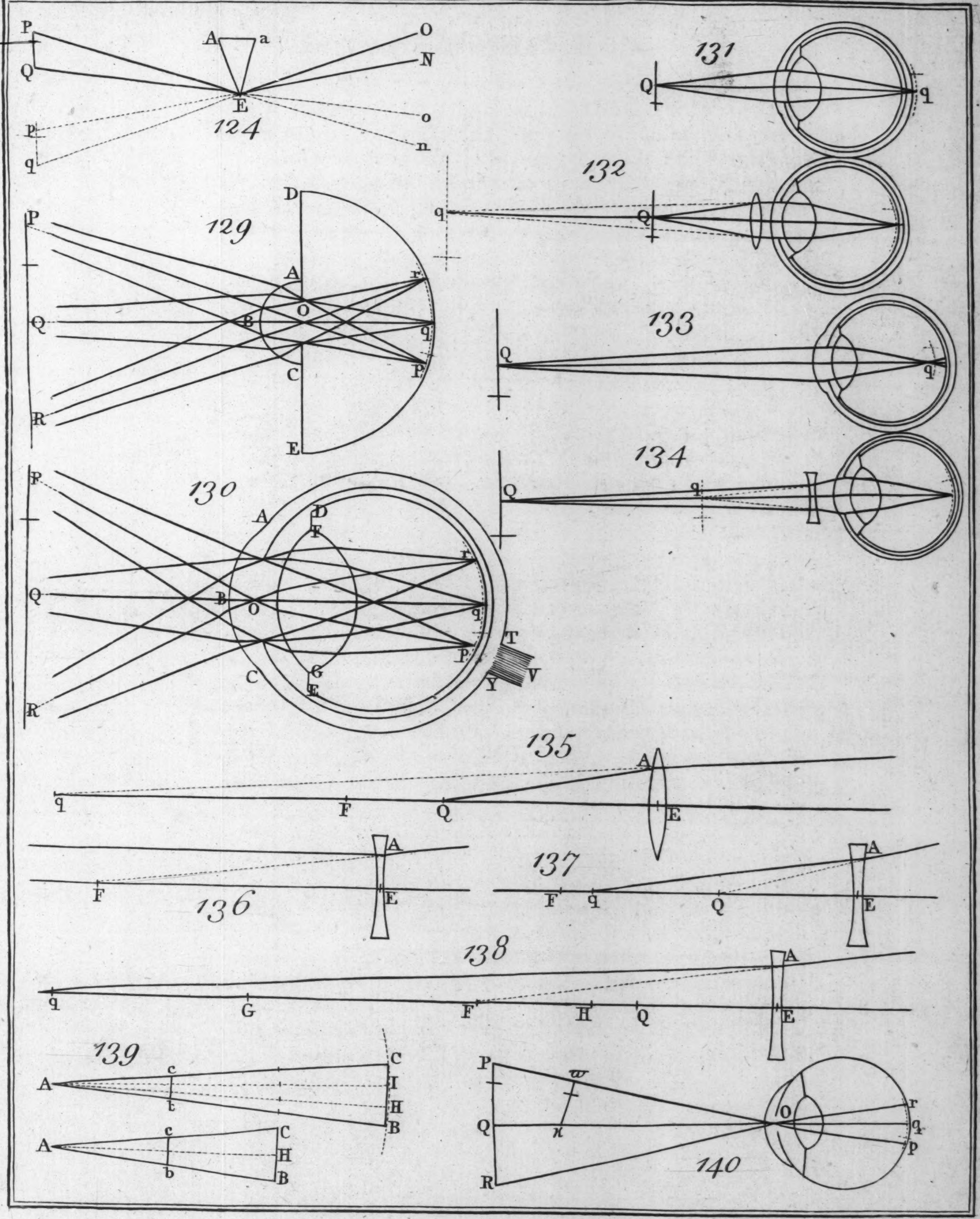
1 Phil. Tranſ. abr. by *Lowth*. Vol. 1. p. 211. and by *Jones* Vol. 4. p. 190.

2 Ibid. *Lowth*. p. 213. and Mem. de l'Acad. Roy. des Scien. ann. 1705. p. 455. 8°.

3 N°. 270. or *Jones*'s abr. Vol. 4. part. 2. p. 1. It. Mem. de l'Acad. an. 1703. p. 233.

4 Animadverſions on Hevelii machina cœleſtis p. 8.

From


124
129
130
131
132
133
134
135
136
137
138
139
140

From whence and from a rule I invented, it appears, that the diameter of the picture of that circle upon the retina was but the 8000th part of an inch at most; and this may be called a sensible point of the retina. That this point is very small any one may perceive from hence that the breadth of the finest hair is visible at the length of one's arm.

143. The apparent magnitude of an object is a quantity of visible extension, measured by, or proportionable to, the angle which the two [a] rays that come from the extremities of the object do make in falling on the eye. For the extremities of the object are seen in the directions of these rays; and in proportion as they make a greater or smaller angle at the eye, the magnitude of the picture upon the retina is longer or shorter [b]; and consequently causes a sensation of a greater or smaller visible extension; consisting of a greater or smaller number of visible points, answering to the number of sensible points of the retina [c], of what magnitude soever these points are supposed to be.

Apparent magnitude to the naked eye defined.

[a] Art. 135.

[b] Art. 136.

[c] Art. 142.

144. The apparent magnitude of any given object is reciprocally as its distance from the eye: that is to say, when the object approaches to the eye, its apparent magnitude increases (in proportion) as its real distance decreases; and on the contrary, it decreases (in proportion) as that distance increases. For the apparent magnitude of an object was defined to be a quantity of visible extension proportionable to the angle which the object subtends at the eye [d], and this angle increases very nearly in proportion as the real distance between the eye and object decreases [e].

How it varies.

[d] Art. 143.

[e] Art. 134.

145. The apparent magnitude of an object seen by the naked eye, in opposition to its apparent magnitude seen through glasses, and for shortness of expression, is often called its true magnitude. And in speaking of the apparent magnitude of an object I always mean the apparent magnitude of its diameter, or of its length or breadth, or of any principal line of it, and not of its surface, or solidity, unless it be particularly specified.

When called true magnitude.

CHAP.

CHAP. X.

CONCERNING VISION WITH GLASSES.

Apparent directions of visible points defined.

146. ANY small object or point of an object, seen by refracted or reflected rays, appears somewhere in the direction of that line, which the visual ray describes after its last refraction or reflection in falling upon the eye.

[a] Art. 18. Fig. 4.

In the experiments to prove the laws of reflection and refraction[a], the pin at *B*, seen by a ray reflected from the water, appeared somewhere in the line *AC* produced, which the visual ray *BCA* described after reflection at *C*, when it advanced to the eye. And as the whole line *CE* appeared lifted up by the refraction at the water, as if it had been a continuation of the line *AC* straight on, so if a straight oar be in part immersed obliquely in water, it appears crooked, as if the part immersed was broken at the surface, and lifted higher. For this part of the oar is seen in the direction of rays which are bent downwards by refraction at their emergence from the water, and consequently advance to the eye as if they came from a place in the water which is higher than the real place of the oar. In like manner any point *P* of an object seen by the ray *PAO* twice refracted, either by passing through the edge of a prism, or of a concave or convex lens, or through the sides of a globe or decanter, or of a drinking glass filled with any transparent liquor; or seen by a ray *PAO* reflected from a plane or spherical looking-glass, appears to the eye at *O*, somewhere in the direction of the last refracted or reflected ray *AO*. Lastly, an object *P* viewed by the eye at *O*, through a multiplying glass, appears at one view in as many different places, *p*, *p* 1, *p* 2, situated in as many different directions *OA*, *OB*, *OC* of the last refracted rays produced, as the glass has different surfaces *DE*, *EF*, *FG* differently inclined to the opposite surface *DH*. For these surfaces, like so many different prisms, give the visual rays *PIAO*, *PKBO*, *PLCO*, so many different bendings at *I* and *A*, *K* and *B*, *L* and *C*, and make them fall upon the eye at *O* in as many different directions *AO*, *BO*, *CO*[b]. And in all these instances when the reflecting or refracting surfaces of the water or glasses are shaken by the wind, or otherwise, the objects seen by reflection or refraction appear to shake and tremble; because the last directions of the visual rays are shaken and varied by those motions.

Fig. 141. to 152.

Fig. 153.

[b] Art. 54.

Now

Now the reason why an object or point of an object appears always in the direction of the last refracted or reflected ray, is, because the place of its picture upon the retina is the same as it would be if the object was really removed from its proper place into the direction of that ray, and was seen by direct rays. And having no sensation of the previous reflections or refractions of the rays at the glasses, but only of their action upon a certain place of the retina, we form the same judgement of the place of the object as we used to do in the more common cases of direct vision.

And determined. Fig. 141. to 153.

It is manifest then, that any point P of an object PQ seen by refractions or reflections, appears somewhere in the line pO, drawn from the corresponding point p of its last image to the eye at any place O. Because all the rays which flowed from P do after the last refraction or reflection flow from or towards the corresponding point p of the last image. The reason why I say the last image will be mentioned in the 155th article.

Their apparent situation determined.

147. It is also manifest why an object seen by refracted or reflected rays appears sometimes upright and sometimes inverted. For when the refracted or reflected rays AO, CO, have the same situation with respect to each other, as two rays that come directly from the same points of the object to the eye, these points will appear in the same situation with respect to each other in both cases [a]. But if the rays that come from these points shall have crossed each other before they arrive at the eye, they will then have a contrary situation to that of two rays coming directly from the same points to the eye; and consequently these two points will appear in the glass in a contrary situation [b]. And one may add that in the former case, the picture upon the retina of the eye will have the same position, though not the same magnitude, as if the glass was removed, and will have a contrary position in the latter case.

[a] Art. 146.
[b] Art. 146.

Apparent magnitude in glasses defined.

148. The apparent magnitude of an object, PQ, seen by refracted or reflected rays either upright or inverted, is a quantity of visible extension, measured by the angle, AOC, which the two [c] rays, AO, CO, that came from its extremities, P, Q, do make, after their last refraction or reflection, in falling on the eye. Or in other words, the object appears greater or smaller in proportion as that angle AOC is greater or smaller. Because its extremities appear in the directions of the last refracted or reflected rays OA, OC [d]; and also because its picture upon the retina is greater or smaller in proportion as these rays constitute a greater or smaller angle at the eye [e].

[c] Art. 135.
[d] Art. 146.
[e] Art. 136.

And determined.

149. Therefore the apparent magnitude of an object, PQ, is also measured by the angle pOq which its last image pq subtends at the

the eye. For the lines AO, pO are but one line continued, and fo are CO, qO, and therefore the angles AOC, pOq are the fame when the image lies before the eye, and are equal when it lies behind it.

How it varies.

150. Hence the apparent magnitude of an object increafes and decreafes in proportion as the eye approaches to or recedes from its laft image, (juft as if it was a real object [a],) placed either before or behind the eye. For when the image is fixed, the angle pOq, when fmall, increafes in the fame proportion as Oq decreafes, and on the contrary [b].

[a] Art. 144.
[b] Art. 134.

When invariable. Fig. 154. to 157.

151. Hence if the laft image be removed to an infinite diftance, that is, if the object be placed in the principal focus of a lens, fphere, or concave looking-glafs [c], its apparent magnitude to the eye at any place whatever will be invariably the fame; and equal to its apparent magnitude feen by the naked eye, fuppofing it put into the place of the center of the fphere, lens, or reflecting concave. For fince all the rays of any one pencil, are parallel to its axis PE, the angle COA, which meafures the apparent magnitude at any point O, is every where equal to the angle QEP at the center E.

[c] Art. 104. 70.

Fig. 158, 159.

The apparent magnitude of the object will alfo be invariable where-ever it be placed, when the eye is fixed at the principal focus of any glafs which makes parallel rays converge to the eye. For conceiving them to flow back again from the eye to the object, they will fall upon the fame points of the object from whence they came while it is moved in any place along the axis of the glafs: and no other rays but thefe can return from the fame points of the object to the eye in that place: therefore the feveral parts of the object will always be feen under the fame angles, and confequently will appear of the fame magnitudes [d].

[d] Art. 148.

Compared to the true magnitude.

152. The apparent magnitude of an object feen by reflected or refracted rays being meafured by the angle which its laft image fubtends at the eye [e], and its apparent magnitude to the naked eye in any place being meafured by the angle which the object itfelf fubtends at the eye in that place [f], it follows that the former apparent magnitude is to the latter, as the former angle to the latter angle. For the meafures of things and the things meafured by them are proportionable.

[e] Art. 149.
[f] Art. 143.

When equal to the true.

153. Confequently the apparent magnitude of an object feen in a glafs, will be equal to its apparent magnitude to the naked eye in the fame place, if the glafs was removed, Firft, when the object touches any thin lens, or any fingle furface. For the image is then equal to the object and coincides with it [g]. Secondly, when the eye touches any thin lens or any reflecting furface. For then the ray

[g] Art. 108.

PAO

PAO will paſs from the object to the eye through the middle of the lens very nearly, and therefore being almoſt ſtraight[a] will make nearly the ſame angle with the axis as an unrefracted ray would do: and when the point of incidence, A, coincides with C at any reflecting ſurface, the incident and reflected rays PA, AO, produced, will alſo make equal angles with the axis or perpendicular QC[b]; and ſo the object will appear under the ſame angle as it would do to the naked eye turned about. Thirdly, when the eye is at the center of a reflecting concave. For then the incident and reflected rays PA, AO will coincide with the direct ray PE[c], and conſequently will make the ſame angles with the axis. Fourthly, when the object is at the center of the reflecting concave. For then the reflected image is alſo at the center and is equal to the object[d]. Fifthly, when a ray coming directly from P to O, would make an angle with the axis equal to the angle AOC, which the refracted or reflected ray PAO makes with it on the other ſide.

[a] Art. 60.
[b] Art. 8.
[c] Art. 10.
[d] Art. 72. 79. Fig. 146. 148. 152.

Leſs than the true through a concave and greater through a convex glaſs.

154. Theſe caſes being excepted the apparent magnitude of an object ſeen through a concave lens is always leſs than the true; and when it is ſeen upright through a convex lens, or a globe, it is greater than the true. For the ray PAO, coming from the extremity of the object to the eye, is bent by the concave lens from its axis, and therefore makes a leſs angle with it at the eye than a ray coming directly from that extremity to the eye. But the ſame ray is bent by the convex lens towards its axis, and therefore makes a greater angle at the eye than the direct ray: and the apparent magnitudes are meaſured by theſe angles.

The whole applied to viſion through any number of glaſſes.

155. What has hitherto been demonſtrated concerning the apparent magnitude of an object PQ, will continue in force if you ſuppoſe the object PQ to be an image formed by another glaſs or other glaſſes. For the rays diverge from either of them in the ſame manner, and for this reaſon I have always called pq the laſt image of the object.

What part of an object is viſible in any glaſs.

156. The place of the eye at O being given, to determine what part of an object is viſible in a given portion or aperture AC of any refracting or reflecting glaſs, draw OA to the edge of the aperture and produce it till it cuts the image in p, and through the center of the glaſs draw pE cutting the object in P; and PQ will be the part in view in the aperture AC. For the whole pencil of rays flowing from P will belong to p after refraction or reflection[e], and conſequently ſome one of thoſe rays will advance to the eye in the line AO drawn through p. If the image be at an infinite diſtance all the rays that belonged to p will be parallel to the axis of the pencil;

[e] Art. 61.
Fig. 154. to 157.

therefore PQ is now determined by drawing EP parallel to OA. In a plane looking-glaſs, pP muſt be drawn from p parallel to qQ, or perpendicular [a] to the glaſs, to cut off the part PQ viſible in the aperture AC. For this glaſs may be conſidered as having a center at an infinite diſtance from it.

Fig. 149.
[a] Art. 66. 78.

How it varies.

157. Hence if the glaſs and object be fixed, the part in view in a given aperture will decreaſe perpetually while the eye recedes from the glaſs; unleſs the image be behind the eye. For then it will decreaſe only till the eye arrives at the image, and after the eye has paſſed by the image it will increaſe perpetually. The reaſon is becauſe the object and image, being fixed in their places, do both increaſe or both decreaſe together, being both terminated by two lines Pp, Qq that meet or croſs in E the center of the glaſs.

When greateſt and leaſt.

158. Therefore the part in view is greateſt when the eye is cloſe to the glaſs, and leaſt when cloſe to the image; and, in this latter caſe, it appears infinitely magnified. For conceiving the diſtance Oq infinitely diminiſhed, the parts pq, PQ cut off by the lines AOp, pEP will both be infinitely diminiſhed; but the magnitude of the angle at O, ſubtended by pq or by AC, continues finite while the angle ſubtended by PQ at O is infinitely diminiſhed: and ſo the diſproportion between theſe angles, that is, between the apparent and true magnitudes of the particle PQ [b] is infinitely great. It appears alſo infinitely confuſed, when the pupil is open, for the reaſon given in the following articles.

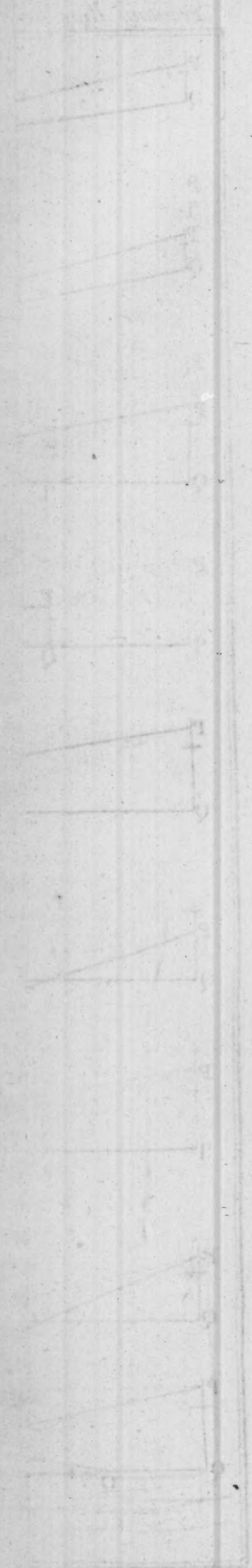
[b] Art. 152.

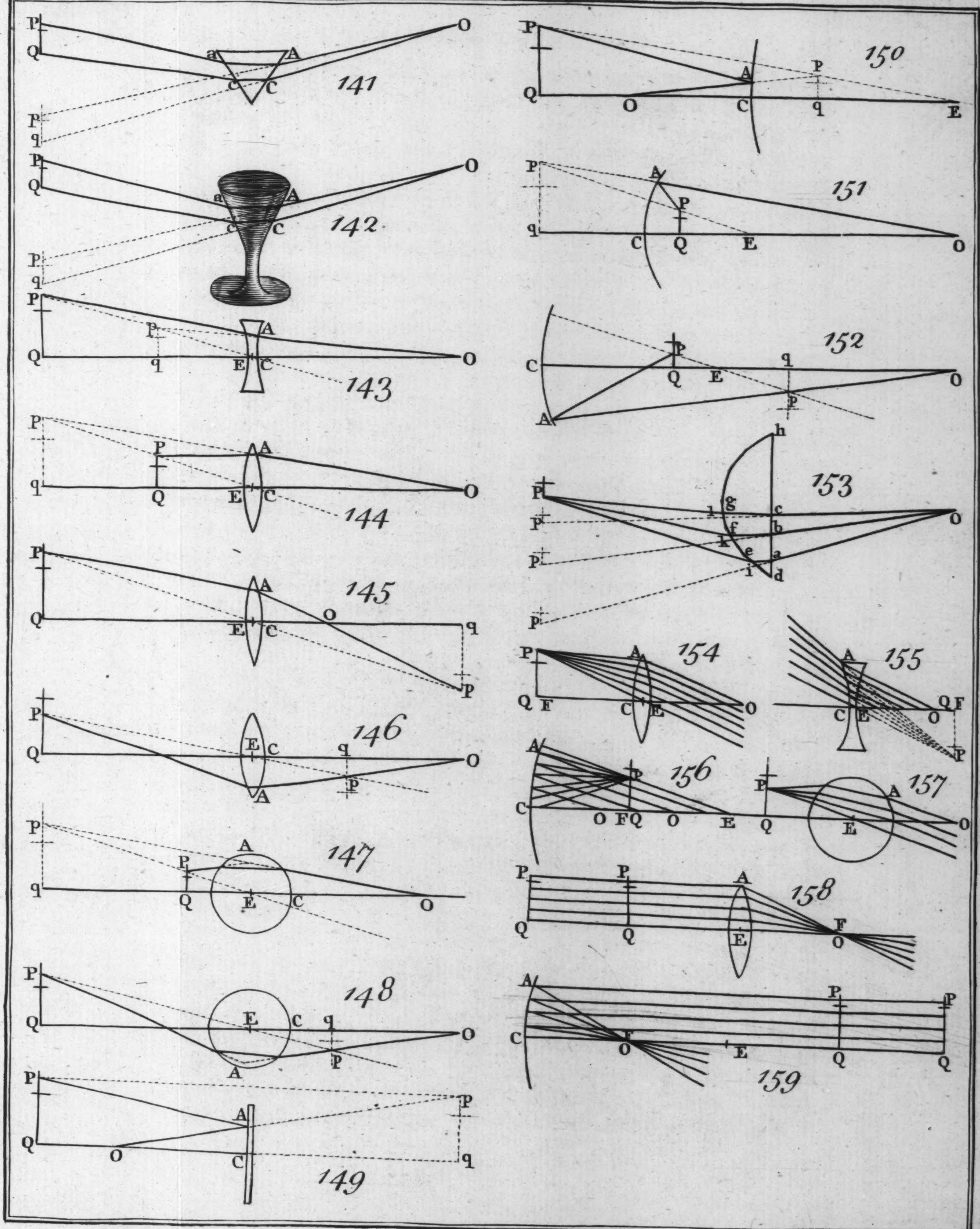
The ſize of a looking-glaſs ſufficient to ſee all ones own body. Fig. 149.
[c] Art. 66.
[d] Art. 24.

159. When a perſon views himſelf in a plane looking-glaſs he appears to himſelf to fill the ſame part of the glaſs wherever he ſtands: and the length and breadth of this part is always half the length and breadth of the correſponding part of his own body. For when O and Q coincide, OC is half of Oq or Qq [c], and therefore AC is half of pq [d] or PQ.

Viſion when confuſed by glaſſes.
[e] Art. 135.

160. Hitherto I have conſidered the pupil of the eye as no bigger than a point, admitting but a ſingle ray from every point of the object [e]; by which means the picture upon the retina would be diſtinct in all caſes. But when the pupil is open, if the image formed by the glaſs be nearer to the eye than the leaſt diſtance at which we can ſee objects diſtinctly with the naked eye, the appearance through the glaſs will be confuſed. Becauſe the rays diverge too much from ſo near an image to be reduced by the eye to a diſtinct picture upon the retina. On the other hand, when the rays converge to an image behind the eye, they will be collected to a diſtinct picture before they arrive at the retina, becauſe the eye is not naturally uſed to conform itſelf to converging rays; and ſo the viſion will




141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159

will be confused in both cases, but may be rendered distinct as followeth.

161. Things which appear confused when seen by direct, refracted or reflected rays may be rendered distinct either by looking through a little hole in a thin plate or bit of paper, or through a convex or concave glass of a proper degree of convexity or concavity; and provided the hole or glass be put close to the eye, the apparent magnitude and situation of the object will be the same in both cases. For if the hole be so small as to admit but a single ray from every distinct point of the object, these rays will fall upon the retina in as many other distinct points, and will make a distinct picture. And when the pencils of rays fall upon a thin lens, their axes go straight through the middle of it[a], and consequently will proceed to the same points upon the retina as when they passed through the hole. Now supposing the lens to have such a figure that the rays of every pencil shall be refracted by it, and by the eye together, to those very points of their axes, which touch the retina, the picture will still be distinct: and will be the same in magnitude and position as before: and the only difference in the effects of the hole and lens will be in the degree of brightness of the picture upon the retina.

How rendered distinct.

[a] Art. 61.

162. A single microscope is only a very small globule of glass, or a small double convex glass, whose focal distance is very short. A minute object pq seen distinctly through a small glass AE by the eye put close to it, appears so much greater than it would to the naked eye, placed at the least distance qL from whence it appears sufficiently distinct, as this latter distance qL is greater than the former qE. For having put your eye close to the glass EA, in order to see as much of the object as possible at one view[b], remove the object pq to and fro till it appears most distinctly, suppose at the distance Eq. Then conceiving the glass AE to be removed, and a thin plate, with a pin-hole in it, to be put in its place, the object will appear distinct, and as large as before[c], when seen through the glass, only not so bright. And in this latter case, it appears so much greater than it does to the naked eye, at the distance qL, either with the pin-hole or without it, as the angle pEq is greater than the angle pLq[d], or as the latter distance qL is greater than the former qE[e].

A single microscope, how much it magnifies.

Fig. 160, 161.

[b] Art. 158.

[c] Art. 161.

[d] Art. 143. 148.

[e] Art. 134.

163. Since the interposition of the glass has no other effect than to render the appearance distinct, by helping the eye to increase the refraction of the rays in each pencil, it is plain that the greater apparent magnitude is intirely owing to a nearer view than could be taken by the naked eye. If the eye be so perfect as to see distinctly by pencils of parallel rays falling upon it, the distance Eq, of the ob-

And in what manner.

object from the glaſs, is then the focal diſtance of the glaſs. Now if the glaſs be a ſmall round globule whoſe diameter is $\frac{1}{15}$ of an inch, ſuch as are eaſily made, its focal diſtance Eq being three quarters of its diameter[a], is $\frac{1}{20}$ of an inch; and if qL be 8 inches, the uſual diſtance at which we view minute objects, this globule will magnify at the rate of 8 to $\frac{1}{20}$ or of 160 to 1.

[a] Art 95.13.

Aſtronomical teleſcope, how much it magnifies, and why. Fig. 162.

164. An aſtronomical teleſcope is compoſed of two convex glaſſes in the following manner. PQ repreſents the ſemidiameter of a remote object, and pq its picture formed by the convex lens L, which being next to the object is called the object-glaſs. In the axis of this glaſs, QLq produced, EA repreſents another glaſs more convex than L, ſo placed, that as qL is the focal diſtance of the glaſs L, ſo qE is the focal diſtance of the glaſs E; and EL the ſum of their focal diſtances. In this ſituation of the glaſſes, I ſay the object will appear to the eye at any point O, diſtinct, inverted and magnified at the rate of qL to qE, that is of the focal diſtance of the object glaſs to the focal diſtance of the eye-glaſs.

For the rays which diverge from the point q of the picture pq, being refracted by the eye-glaſs, will emerge upon the eye at O in lines parallel to the axis qEO; becauſe qE is ſuppoſed to be the focal diſtance of the eye-glaſs; and for the ſame reaſon, the rays which diverge from any collateral point p, of that picture pq, will emerge from the eye-glaſs, after refractions at A, in lines parallel to the line or ray pE; this line being the axis of an oblique pencil of rays, part of which diverge from p upon the glaſs. An eye therefore which can ſee diſtinctly by pencils of parallel rays being placed any where at O *, among the interſections of theſe pencils, will ſee the points of the object diſtinctly.

Now to the eye at O the apparent magnitude of the picture pq, or object PQ, is meaſured by the angle EOA[b], or by the equal an-

[b] Art. 148.

Fig. 162.

* PROPOSITION. *The point* O, *where the axes* LB, LA *of the extreme pencils croſs the axis* LE, *is a little beyond the principal focus of the eye-glaſs.*

For we may ſuppoſe all the axes of the ſeveral pencils to proceed from the point L as from a focus of incidence; and their focus after refraction may be found by article 104. But the firſt term Lq, in the rule there laid down, is conſiderably greater than the ſecond qE, by conſtruction: wherefore the fourth term, that is, the diſtance of O from the other principal focus is but ſmall, when compared with the focal length of the eye-glaſs. *Q. E. D.*

N. B. The point O, thus determined, is the place from which an eye can ſee the moſt poſſible of an object, through a given teleſcope.

The ſituation of common eyes in a double microſcope is determined after the ſame manner.

Hugens's Dioptr.

gle

gle qEp; but to the naked eye at L, if the glaſs was removed, the apparent magnitude of the object is meaſured by the angle QLP, or by the equal angle qLp; the oblique axis PLp being ſtraight [a]. Therefore the former apparent magnitude is to the latter, as the angle qEp, to the angle qLp; and conſequently as the latter diſtance qL, to the former qE [b].

[a] Art. 61.

[b] Art. 134.

165. The object which appeared inverted in the former teleſcope [c], will appear upright and diſtinct through two more convex eye-glaſſes ſubjoined to it; at a diſtance from each other, equal to the ſum of their focal diſtances; and when their focal diſtances are equal to each other, the object will be magnified juſt as much as it was before. For the pencils of parallel rays EOF, AOB, &c, which are continued to the glaſs FB, will be formed by it into a ſecond image $\varpi\kappa$; and the focus ϖ, of any oblique pencil OB, will be determined by the interſection of the line $\varpi\kappa$, perpendicular to the common axis of the glaſſes, and of the oblique axis $F\varpi$, drawn parallel to the incident rays OB [d]. This point ϖ being the focus of incident rays on the laſt glaſs GC, the emergent rays CD will be parallel to their oblique axis ϖG; becauſe the rays that flow from κ are ſuppoſed to emerge parallel to the direct axis. Therefore to the eye at D, where theſe emergent pencils croſs, the object will appear diſtinct, and upright [e]. And when the glaſſes F and G are exactly equal, the image $\varpi\kappa$ will be exactly in the middle between them; and ſo the triangles $\varpi F\kappa$, $\varpi G\kappa$ will be exactly equal. Conſequently the angle CDG, which now meaſures the apparent magnitude to the eye at D, will be equal to the angle $\varpi G\kappa$ or $\varpi F\kappa$ or BOF or AOE, which meaſured it before to the eye at O.

A teleſcope made of four convex glaſſes conſidered. Fig. 163.

[c] Art. 147.

[d] Art. 116.

[e] Art. 147.

166. In a teleſcope of a given length the quantity of objects taken in at one view, depends upon the breadth of the eye-glaſs. For as AE is greater or ſmaller, the angle ALE or its equal PLQ is alſo greater or ſmaller; and this angle takes in all the objects that can be ſeen at one view on one ſide of the axis of the teleſcope.

How much they take in at one view. Fig. 162, 163.

167. The difference between the aſtronomical teleſcope and *Galileo*'s teleſcope or a common perſpective-glaſs is this; inſtead of the convex eye-glaſs placed behind the image to make the rays of each pencil go parallel to the eye, there is placed a concave eye-glaſs AE as much before it; which opens the rays of each pencil that converged to q and p, and makes them emerge parallel upon the eye; as is evident by conceiving the rays to go back again through the eye-glaſs, whoſe focal diſtance we ſuppoſed was Eq. The eye muſt be put cloſe to the glaſs to receive as many pencils as poſſible; and then, ſuppoſing an emerging ray of an oblique pencil produced backward along

Galileo's teleſcope conſidered. Fig. 164.

along AO, the apparent magnitude of the object is measured by the angle AOE[a] or its equal qEp, which is to the angle qLp (or QLP, the measure of the true magnitude,) as qL to qE, as before in the other telescope. It is manifest, by the 147th article, that objects in this telescope appear upright.

[a] Art. 148.

This takes in less than the former.

168. The quantity of objects taken in at one view in this telescope does not depend upon the breadth of the eye-glass, as in the astronomical telescope, but upon the breadth of the pupil of the eye. Because the pupil is less than the eye-glass, and the lateral pencils do not now converge to, but diverge from the axis of the glasses. Upon this account the view being narrower is not so pleasant as in the former telescope.

Sir *Is. Newton*'s reflecting telescope. Fig. 165.

169. Sir *Isaac Newton*'s reflecting telescope magnifies the diameter of a remote object in the proportion of the focal distance of the reflecting concave to the focal distance of the convex eye-glass, and shews it inverted. Let ST be an image of a remote object PQ formed by reflections from a large concave surface AC, and terminated by the lines $PESA$, $QETC$ drawn through its center E. Now because this image cannot be viewed through an eye-glass placed directly before it (for then the spectator would intercept the rays that are coming to the concave) therefore let the several pencils of rays which converge towards it in coming from the broad concave AC, be reflected sideways from a small polished plane, represented by ac; and then the second image st, formed by this plane, will be equal to the first image ST[b]. Let tl be the focal distance of a small convex eye-glass kl and the rays which flow from any point s will be refracted through this glass, to the eye at o, in the lines ko drawn parallel to the oblique axis sl; and so the apparent magnitude of the object, PQ, to the eye at o, will be measured by the angle kol or slt[c]: but to the naked eye at E, it is measured by the angle PEQ or SET. Therefore the former apparent magnitude is to the latter, as the angle slt to the angle SET or, (because their subtenses st, ST are equal,) as ET to lt[d] or as CT to lt, when the object is remote[e]. Note that the plane acb is much too broad in comparison to the concave ACB, which could not be helped in so small a draught. That the appearance of the object is inverted or turned from right to left, is evident by the 147th article.

[b] Art. 66. 78.
[c] Art. 148.
[d] Art. 134.
[e] Art. 69.

Why so much shorter than others.

170. Dioptrick telescopes which magnify much being very long and troublesome to be managed, Sir *Isaac Newton* proposed this method to shorten telescopes[1]; which answers to admiration; as appears by a table in the 12th chapter, of the lengths of both sorts of

1 Opticks, p. 95.

telescopes

telescopes which magnify equally with equal distinctness. The reason why dioptrick telescopes cannot be shortened as much as these, and still magnify as much, by diminishing the focal distances of the eye-glasses[a], in short is this. The images made by refractions through the convex object-glasses, being much more imperfect than those which are made by reflections from concave surfaces, will not bear to be magnified so much by so small eye-glasses[b], without appearing confused: and the chief cause of those imperfections in the pictures is the unequal refrangibility of rays of different colours[c].

[a] Art. 164.
[b] Art. 162.
[c] Art. 220.

171. The following description of Mr. *Gregorie*'s reflecting telescope differs from the author's chiefly in this; that he directs his larger reflecting concave to be made of a parabolick figure, and his lesser of an elliptical one, instead of the spherical surfaces now used; which are the only figures that can be polished without insuperable difficulties.

Mr. *Gregorie*'s reflecting telescope described.
Fig. 166.

It is proposed to make a reflecting telescope with two concave metals and a convex eye-glass and to shew its effects. Let the given focal distances of the lesser and the larger concave and of the convex eye-glass, be equal respectively to the lines t, T, q; and in a given line $ctqCl$, designed for their common axis, take in one and the same direction, $ct = t$, $tq = T$, $qC = \frac{t \times t}{T}$ and $ql = q$; and place the eye-glass at l, the lesser concave at c, and the larger at C; so that their concavities may respect each other; and let the incident rays, as QA, QB, be reflected from the larger to the lesser concave, and from thence to the larger again, where let them pass through a moderate hole made in the middle of it at C, and then be refracted through the eye-glass kl to the eye at o; I say a remote object will appear distinct and upright and magnified in the ratio of $T \times T$ to $t \times q$; that is, of the square of the focal distance of the larger concave, to the rectangle under the focal distances of the lesser concave and of the eye-glass.

172. For a pencil of rays QA, QB coming parallel to the common axis, will be reflected from the larger concave ACB to its principal focus T; where crossing one another, and falling upon the lesser concave acb, they will be reflected from it to the point q. For since the focal distance $TC = T = tq$ by construction; by taking away the common part Tq, we have $tT = qC = \frac{t \times t}{T}$ by construction; that is, we have tT, tc, tq continual proportionals, as they should be[d]; and since pl is the focal distance of the eye-glass kl, the rays

[d] Art. 71.

rays that flow from q will emerge from it in parallel lines, and therefore will produce a diſtinct appearance of the remote point Q from which they came.

Fig. 167.

173. Let ST be the image of the object PQ formed by reflection from the large concave; and it will be terminated by the line PES, drawn through E, the center of this concave, parallel to the rays PA, PA that flow from P[a]. Again, the rays that flow from this image ST, will be reflected from the leſſer concave and form a ſecond image pq; which will be terminated by the line Sep drawn through the center e of this concave[b]; and the rays that diverge from p will emerge from the eye-glaſs kl in the lines ko parallel to the line pl, drawn through the center of the eye-glaſs[c]. Therefore the object PQ will appear upright, becauſe the rays ko lye on the ſame ſide of the common axis Qlo as the point P from which they came.

[a] Art. 79.
[b] Art. 79.
[c] Art. 98.

174. In the ſecond image pq take a line qs equal to the firſt image TS; and if the image pq was equal to qs, the object would appear through the eye-glaſs under an angle equal to qls[d]; which is to the angle PEQ or SET, under which it appears to the naked eye at E, as TE or TC to ql[e]; and ſo the object would be magnified in the ſame ratio as in Sir *Iſaac Newton*'s teleſcope. But ſince the triangles epq, eST are ſimilar; and ſince we had tq to te (as te to tT[f], and disjointly as eq to eT, that is,) as pq to ST or qs; it appears that pq is bigger than qs, and alſo the viſual angle kol or plq bigger than qls, in the ſaid ratio of tq to te. And ſo the object being farther magnified in this ratio of tq to te or, by conſtruction, of TC to tc, is magnified in the whole in the compound ratio of TC to tc, and of TC to ql, that is in the ratio of TC ſquare, to the rectangle under tc and ql.

[d] Art. 151. 155.
[e] Art. 134.
[f] Art. 172.

To adapt it to a near object.

175. For viewing near objects the little concave muſt be removed a little from the large one. Becauſe while a remote object approaches, its image TS will alſo approach towards t[g]; and while tT is diminiſhed, its reciprocal tq will be increaſed[h].

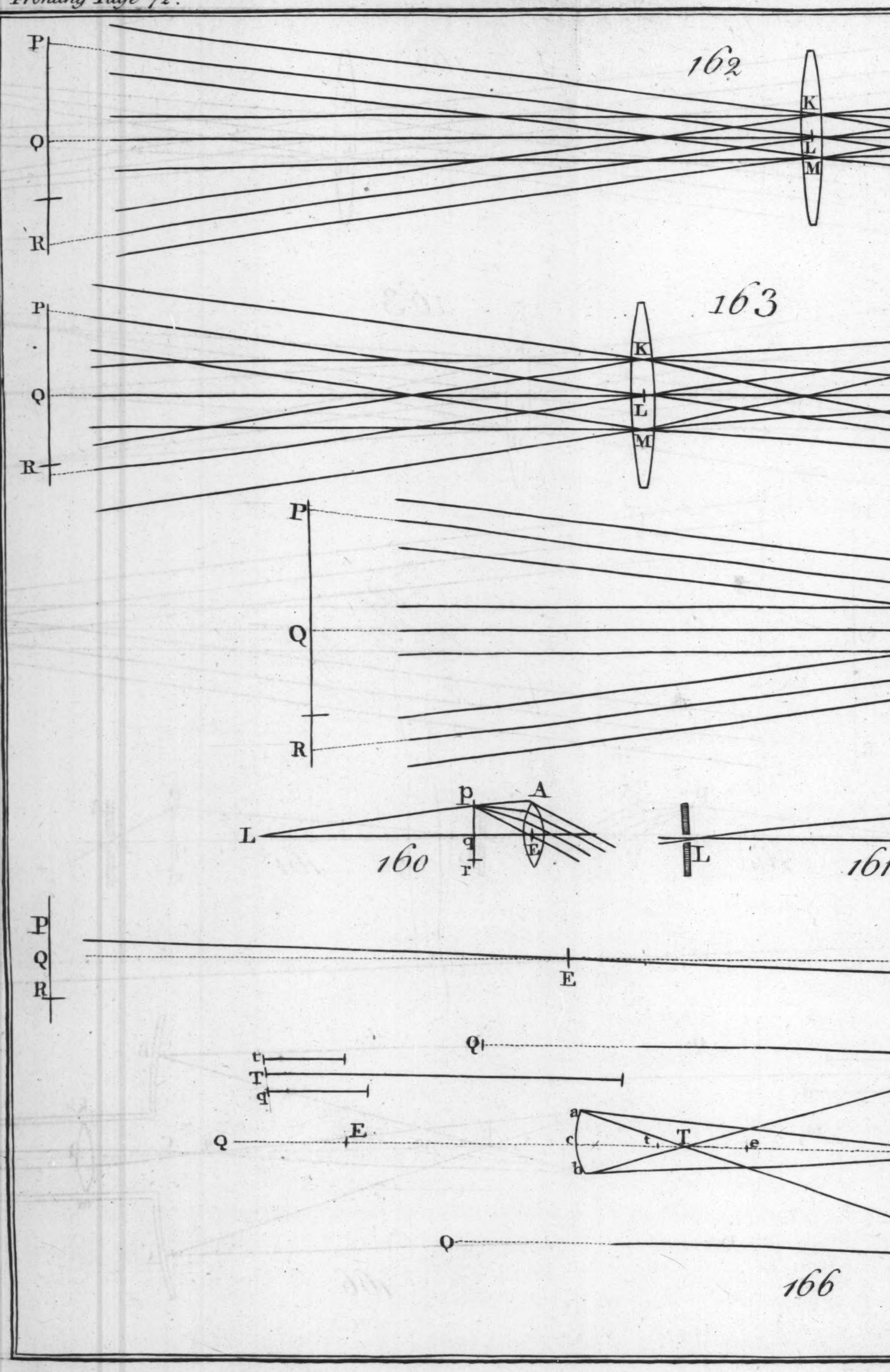
[g] Art. 72.
[h] Art. 172.

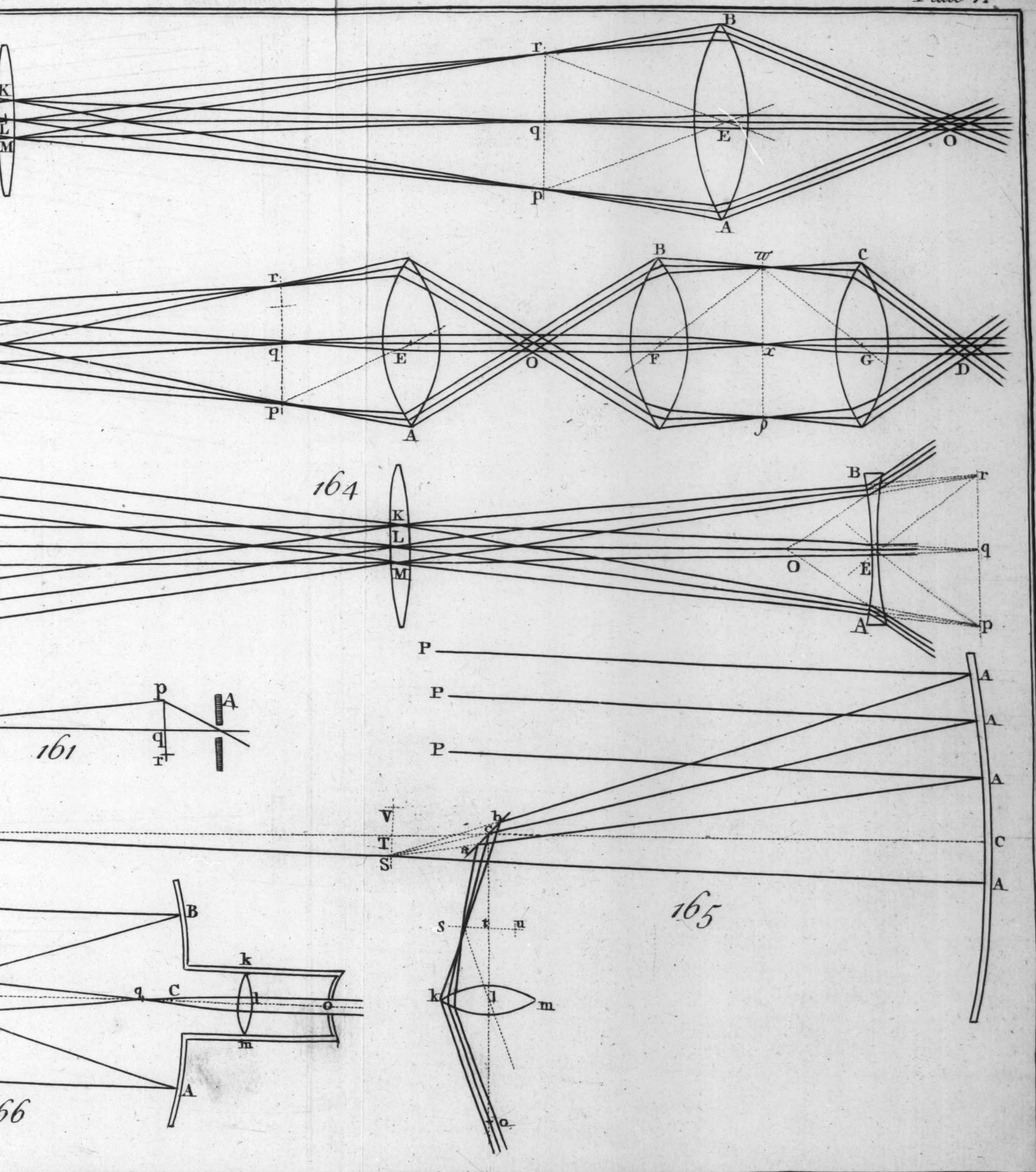
And to ſhort-ſighted eyes.

176. Therefore to fit this teleſcope for a ſhort-ſighted perſon, ſince the eye-glaſs is uſually fixt, the little ſpeculum muſt be moved ſomewhat nearer to the large one. For then the interval tT will alſo be diminiſhed and its reciprocal tq will be increaſed; and ſo the rays will fall upon the eye-glaſs diverging from a nearer point than its focal diſtance; and conſequently will emerge from it diverging upon the eye.

A more general rule for the magnifying power.

177. By a farther contraction of the interval between the concaves, the image pq may be projected through the hole in the large concave,


162
P
Q
R
K
L
M
163
P
Q
R
K
L
M
P
Q
R
p
A
q
r
E
L
160
L
P
Q
R
E
Q
t
T
q
Q
E
a
c
b
t
T
e
Q
166


164
161
165
66

concave, to any given place behind it; and by removing the eye-glaſs to the ſame diſtance from the images as before, the viſion would become diſtinct again; and the object would be more magnified than before, as much as the ratio of tq to te or tc is made bigger than the ratio of TC to tc; as appears by the demonſtration [a]. But by enlarging the image pq, it becomes more obſcure and imperfect, and conſequently the appearance of the object leſs bright and diſtinct. Beſides, as the image becomes larger, the leſs of it, and of the object, can be ſeen at one view through a given eye-glaſs.

[a] Art. 174.

The viſible area is as the breadth of the eye-glaſs.

178. All things being fixt in their places, the diameter of an object taken in at one view, is proportionable to the breadth of the eye-glaſs, if the hole in the large concave does not limit it. For the angle of reflection pce, at the middle point of the leſſer concave, being equal to the angle of incidence ecS; it appears, that while pq and kl are increaſed or diminiſhed in any ratio, the image ST and the object PQ will alſo be increaſed or diminiſhed in the ſame ratio.

Is enlarged by two eye-glaſſes.

179. Now if an eye-glaſs of a given focal diſtance and convexity, be made very broad, it will become too thick; and ſo the rays will fall too obliquely upon one or both its ſurfaces near the margin of it; and this obliquity will cauſe too many of them to be reflected, and the reſt that are tranſmitted, to be too much refracted, in compariſon to thoſe pencils that paſs through the middle of the ſaid lens. Therefore to increaſe the viſible area of the object, it is neceſſary to project the image pq two or three inches beyond the hole in the large concave, and to intercept the rays that are tending towards it, with a thinner and broader convex glaſs fg put cloſe to the backſide of this concave; which glaſs will cauſe the rays to converge quicker than before, and to form an image vx nearer to it, and ſmaller, than pq; both being terminated by a line pvg drawn through the center of this glaſs. And then the rays of each pencil diverging from this new image vx, muſt be received by another convex eye-glaſs hi, that ſhall make them emerge towards the eye in parallel lines. A meniſcus glaſs, whoſe convex ſide is placed towards the converging pencils fvh, is fitteſt for this purpoſe; becauſe the rays will paſs through its edges leſs obliquely, than through a glaſs of any other ſhape.

Fig. 168, 169.

The eye-ſtops.

180. To prevent collateral rays, that paſs by the ſides of the ſmaller concave, through the hole in the larger, and thoſe alſo which are reflected from the imperfect margins of them both, from entering into the eye; it is neceſſary to place a thin plate with a proper hole in it to circumſcribe the image at x, and alſo another very ſmall

hole at o, where all the pencils croſs one another immediately before they enter the eye. The breadth of this latter hole muſt be no bigger than that of the principal pencil at o, and the places of them both muſt be exactly adjuſted; otherwiſe the teleſcope can have no good effect.

The little concave ſpeculum may be changed for a convex one. Fig. 170.

181. Teleſcopes of this kind are ſometimes made with a little convex ſpeculum inſtead of the concave one. If their focal diſtances be equal, and the vertex of the convex de, be placed at e, where the center of the concave was, the teleſcope will magnify in the ſame ratio as before; but will ſhew the object inverted; unleſs it be ſet upright by three convex eye-glaſſes, as in a dioptrick teleſcope. For a pencil of rays converging from the large concave towards its focus T, being intercepted by the little convex de, will be reflected by it to the ſame point q as before by the little concave bc. For the point t being the principal focus of both theſe little ſpeculums, we have tT, te (or tc) and tq continual proportionals as before[a]. Through any point S of the firſt image ST and through the center e of the little concave, draw Sep terminating the image pq formed by this concave[b]; in like manner through c the center of the little convex de, and through the ſame point S, draw cSr terminating the image qr formed by this convex. Theſe images qp, qr lye on contrary ſides of the axis, and therefore the object appears in contrary poſitions. But theſe images are equal, and of conſequence the object appears equally magnified. For we have $tq : te :: te : tT :: tq \mp te : te \mp tT$, that is $:: eq : eT :: cq : cT$. And the triangles peq, TeS being ſimilar, and alſo qcr, Tcs, we have $pq : ST$ ($:: eq : eT :: cq : cT ::$) $qr : ST$; therefore pq is equal to qr.

[a] Art. 71.

[b] Art. 79.

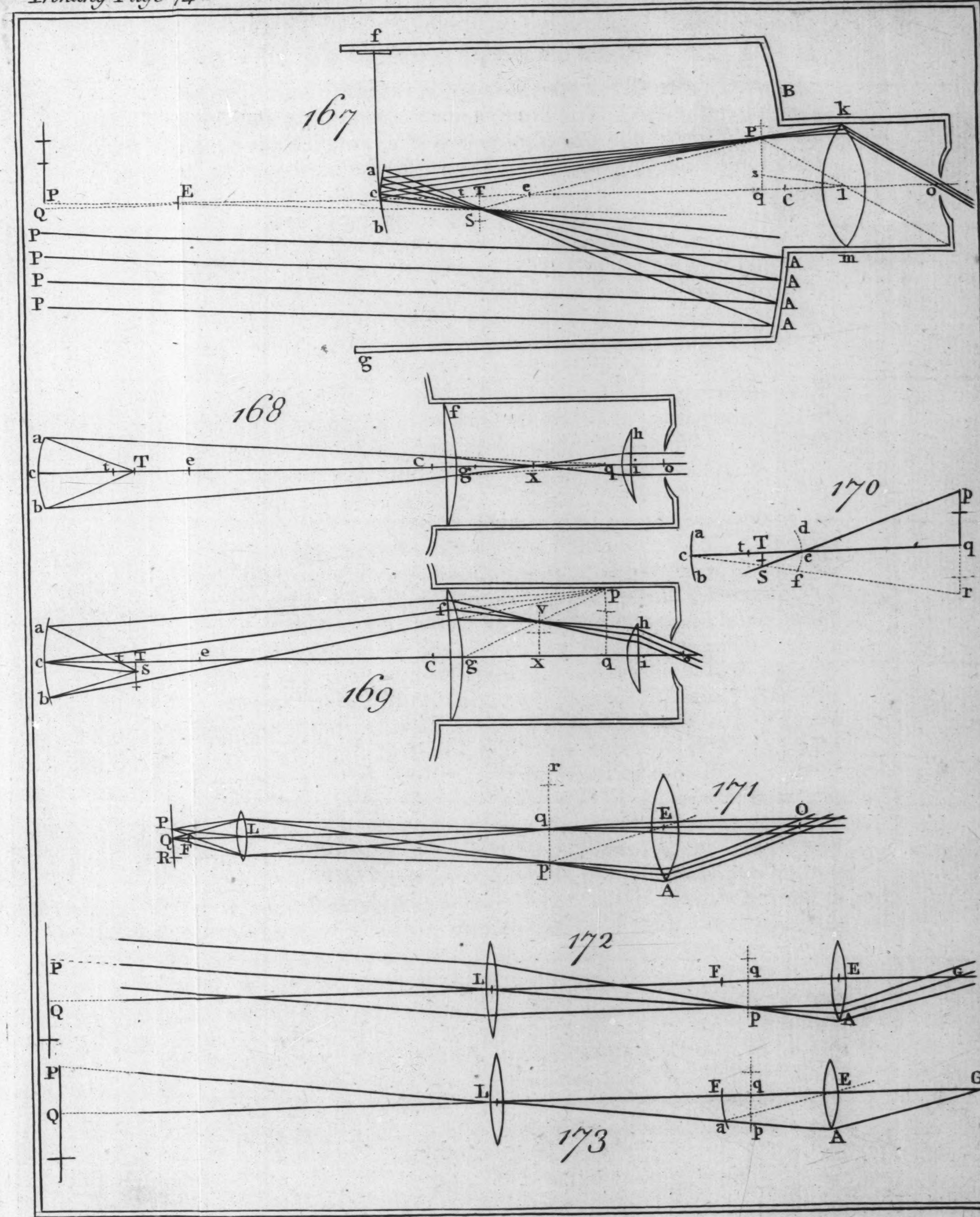
Double microſcope conſidered. Fig. 171.

182. A double microſcope is compoſed of two convex glaſſes placed at E and L. The glaſs L next the object PQ is very ſmall and very much convex, and conſequently its focal diſtance LF is very ſhort; the diſtance LQ of the ſmall object PQ is but a little greater than LF; ſo that the image pq may be formed at a great diſtance from the glaſs[c], and conſequently may be much greater than the object itſelf[d]. This picture pq being viewed through a convex eye-glaſs AE, whoſe focal diſtance is qE, appears diſtinct as in a teleſcope. Now the object appears magnified upon two accounts; firſt becauſe if we viewed its picture pq, with the naked eye, it would appear as much greater than the object, at the ſame diſtance, as it really is greater than the object, or as much as Lq is greater than LQ[e]; and ſecondly becauſe this picture appears magnified through the eye-glaſs as much as the leaſt diſtance at which it can be ſeen diſtinctly with the naked eye, is greater than qE, the focal

[c] Art. 108.

[d] Art. 115.

[e] Art. 115.


167
168
169
170
171
172
173

focal distance of the eye-glass [a]. For example, if this latter ratio be 5 to 1, and the former ratio of Lq to LQ be 20 to 1, then upon both accounts the object will appear 5 times 20, or 100 times greater than to the naked eye.

[a] Art. 162.

183. To fit these telescopes and microscopes to short-sighted eyes, the glasses E and L must be placed a little nearer together; so that the rays of each pencil may not emerge parallel but may fall diverging upon the eye [b]; and then the apparent magnitude will be altered a little but scarce sensibly; as is demonstrated in the next article.

To fit telescopes and microscopes to defective eyes.

[b] Art. 104. 108. 111.

184. Suppose the interval LE between the two convex glasses to be greater or less than the sum of their focal lengths, and let EF be the focal distance of the eye-glass, and Lq that of the object-glass; I say the apparent magnitude will be to the true as LF to FE, that is, as the interval of the glasses diminished by the focal distance of the eye-glass, to the focal distance of the eye-glass. For the axes of all the pencils which pass through L, as PLA, will be refracted by the eye-glass to a focus G, where the eye being placed will see the whole object PQ, though the aperture of the pupil and of the object-glass be never so small [c]; and the object PQ will appear under the angle AGE. But L being a focus of incident rays upon the eye-glass, we have $LF : LE :: LE : LG$ [d], and disjointly $LF : FE ::$ ($LE : EG ::$ the angle EGA, to the angle ELA [e] or PLQ [f]) as the apparent magnitude to the true.

A more general demonstration of telescopes. Fig. 172, 173, 174, 175.

[c] See note to Art. 164.

[d] Art. 107.

[e] Art. 134.

[f] Art. 61.

185. Hence according as the interval of the glasses is greater or less than the sum of their focal distances, the apparent magnitude is to the true, in a greater or less proportion than that of the focal distances.

186. The brightness of the appearance through a given telescope or microscope is more or less in proportion to the aperture of the object-glass. For supposing it covered with paper, all but a small hole in the middle, the magnitudes of the pictures pq in the focus of the glasses, and of that upon the retina would not be altered; but the hole at L being smaller than before, there are fewer rays in every pencil, and consequently in every point of those pictures, and so they appear more obscure. If the aperture and object-glass remain the same, things appear brighter or fainter according as the focal distance of the eye-glass is longer or shorter; that is, according as the telescope or microscope magnifies less or more [g]. For the same quantity of light spread over a smaller or larger picture or part of the retina will make it brighter or duller.

The apparent brightness through them.

[g] Art. 164. 182.

 187. Hitherto

All appearances the same when the eye is out of the axis of the glass or glasses. Fig. 176.

187. Hitherto I have supposed the eye to be always placed at some point O in the common axis of the refracting or reflecting surfaces. Now let it be placed at any point o in a line Oo perpendicular to the axis Qq; I say that all the appearances will be the same or at least not sensibly different from what they were before. For let pq be the last image of an object, and PQ the last but one, or the object itself; draw the lines po, qo meeting the next surface in a and c; and the points P and Q will appear to the eye at o in the directions of those lines oa, oc. Whence drawing pO meeting the surface in A; since the directions OA, oa, in which P is seen, lye the same way from the directions OC, oc, in which Q is seen, it is evident that the apparent situation of the extremities P, Q is the same at both places of the eye; and also the apparent magnitude, which is measured by the angle aoc[a] or poq or pOq or AOC. For the small angles poq, pOq, being subtended by the same image pq, very nearly at equal distances po, qO from o and O, are very nearly equal. The apparent brightness of the object is also the same; because the density of the rays, that enter the pupil, at any part of the perpendicular plane represented by Oo, is nearly the same[b]. For the rays flow from or towards the last image pq just as if it was a luminous body. And lastly the degree of apparent distinctness or confusion is the same also, because the angles which the pupil, placed at O or at o, subtends at p and q, or the mutual inclinations of the rays in each pencil are very nearly equal.

[a] Art. 148.

[b] Art. 25.

A general observation upon vision.

188. This general observation upon vision is worth remembering. That the apparent distinctness and confusion of an object depends upon the mutual inclination of the rays to each other in any one pencil when they fall upon the eye[c]; the apparent magnitude, upon the inclination of the rays of different pencils to each other when they fall upon the eye[d]; the apparent situation, upon the real situation of the extream pencils when they fall upon the eye[e]; and the apparent brightness and obscurity, upon the quantity of rays in every pencil.

[c] Art. 160.

[d] Art. 148.

[e] Art. 147.

The portable *camera obscura*. Fig 177.

189. The portable *camera obscura*, or dark chamber, commonly sold in the shops, will require but a short description. The theory is this; the rays that come from the object PQR after passing the lens E are tending to form an image pqr; but being reflected upwards by the looking-glass ABC, they form an horizontal image $\varpi\kappa\varrho$ upon a glass plane, whose unpolished side lyes uppermost; upon which a copy of the picture may be sketched out with a black lead pencil; and to the spectator facing the object, the picture appears upright.

upright. This figure repreſents a ſection of the machine through the axis of the tube that holds the lens, and through the middle of the ſquare box and the looking-glaſs within it. The ſection of the ſide oppoſite to the tube is not here repreſented, it being a door that opens ſideways; the edges of the rough glaſs at the top are placed in two grooves upon the ſides of the box; and being taken off, it is placed in a drawer *ef* at the bottom of the box; the looking-glaſs *ABC* may alſo be drawn out of the grooves in the ſides of the box and lodged in the ſame drawer. The ſquare wooden tube conſiſts of 3 parts; the innermoſt that carries the lens, draws outwards or inwards to make the pictures diſtinct. The parts *gh* and *ik*, being fixt together and to the box with ſmall bolts, may be taken aſunder and put into the box; then the lid *at* at the top, and the door at the end, being both ſhut and fixt, the machine becomes more commodious for carriage. The inſide of the lid whoſe ſection is *at*, has two wings, that open to right angles on each ſide of it, and reſt upon the ſides of the box, to ſhade the image upon the rough glaſs.

190. The conſtruction of the magick lantern is briefly this; *ABCD* is a tin lantern, from whoſe ſide there proceeds a ſquare or round arm or tube *bnkclm*, conſiſting of two parts; the outermoſt whereof *nklm* ſlides over the other, ſo as that the whole tube may be lengthened or ſhortened thereby. In the end of the arm *nklm* is fixt a convex glaſs *kl*: about *de* there is a contrivance for admitting and placing an object *de* painted in dilute and tranſparent colours on a plane thin glaſs; which object is there to be placed inverted. This is uſually ſome ludicrous or frightful repreſentation, the more to divert the ſpectators: *bhc* is a deep convex glaſs, ſo placed in the other end of the prominent tube, that it may ſtrongly caſt the light of the flame *a* on the picture *de* painted on the plane thin glaſs. And here it is to be noted, that the glaſs *bhc* is only deſigned for the ſtrong illumination of the picture *de*, and has nothing to do in the repreſentation; and therefore in ſome of theſe lanterns, inſtead of the glaſs *bhc*, we ſhall find a concave ſpeculum ſo placed, that it may ſtrongly caſt the light of the flame *a* on the picture at *de*; and ſometimes both are uſed.

Deſcription of the magick lantern by Mr. *Molyneux*. Fig. 178.

191. Wherefore let us now conſider the picture *de* as a very lightſome object of diſtinct colour and parts. And let us conceive *de* more remote from the glaſs *kl* than its focus. It is then manifeſt that the diſtinct image of the object *de*, ſhall be projected by the glaſs *kl* on the oppoſite white wall *FH* at *fg*; and here it ſhall be repreſented erect. For now the whole chamber *EFGH* is dark, the lantern

lantern $ABCD$ inclosing all the light; so that in effect this appearance of the magick lantern is no more than what we are told concerning the representation of outward objects in a dark room by a convex glass; and here we may observe, that if the tube be contracted, and thereby the glass kl brought nigher the object de, the representation fg shall be projected so much the larger; and so much the more distant from the glass kl; according to the rules before laid down[a]. So that the smallest picture at de may be projected at fg in any greater proportion required, within due limits. From whence the name of Lanterna Megalographica. And consequently, protracting the tube and drawing the glass kl more distant from the object de, will diminish the representation fg, and project it nigher the glass kl.

[a] Art. 104. 108. 115.

A comparison of different ways of illuminating microscopical objects, pictures in a magick lantern, &c. Fig. 179, 180, 181.

192. Of a luminous object QR let qr be the image formed by reflection from a concave surface, or by refraction through a convex lens, or sphere AC; whose center is E, principal focus F, axis $QEFC$, and semiaperture AC; and let a perpendicular FG, to the axis, cut the outermost ray QA in G; I say the brightness of the several pictures qr, will be very nearly as $\overline{FG}^2$ directly and $\overline{FE}^2$ inversely.

For, not regarding the small losses of light by the several reflections and refractions, the quantity collected to the point q is very nearly as $\frac{\overline{AC}^2}{\overline{CQ}^2}$[b], and consequently the quantity in the area of the whole picture qr, as $\frac{\overline{AC}^2}{\overline{CQ}^2} \times \overline{QR}^2$ or $\frac{\overline{FG}^2}{\overline{FQ}^2} \times \overline{QR}^2$. But the area of the picture is as $qr^2 = \frac{\overline{Eq}^2}{\overline{EQ}^2} \times \overline{QR}^2 = \frac{\overline{FE}^2}{\overline{FQ}^2} \times \overline{QR}^2$. Because, in the reflecting concave, we have $Fq : FE :: FE : FQ$[c]; and consequently $Eq : EQ :: FE :: FQ$; and in the lens and sphere we have $Qq : QE :: QE : QF$[d], and consequently $Eq : EQ :: FE : FQ$. Therefore the brightness of the picture, or the density of the rays in its area, being as their quantity directly and the area inversely, is as $\frac{\overline{FG}^2}{\overline{FE}^2}$ very nearly; and the more exactly as the aperture is smaller and the object farther off.

[b] Art. 25.

[c] Art. 71.

[d] Art. 107.

193. *Corol.* 1. In a given speculum, lens or sphere, the brightness of the picture of a given object, is as $\overline{FG}^2$; and therefore increases continually with the distance of the luminous object from the focus F.

194. *Corol.*

194. *Corol.* 2. If the luminous object be very remote, and the apertures of ſeveral ſpecula, lenſes and ſpheres be equal to one another, the degrees of brightneſs of the ſeveral pictures formed by them are reciprocally as the ſquares of their reſpective focal diſtances very nearly.

195. *Corol.* 3. Conſequently if the ſeveral apertures be equal portions of equal ſpheres, the degrees of brightneſs of the ſeveral pictures formed by a concave ſpeculum, a double-convex glaſs, a glaſs ſphere and a plano-convex glaſs, are reſpectively as the ſquares of the decreaſing muſical progreſſion 12, 6, 4, 3. Becauſe the reſpective focal diſtances are $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ of the diameter of the given ſphere, by Art. 69, 103, 95; and the reciprocals of an arithmetical progreſſion are called a muſical progreſſion.

196. *Corol.* 4. Therefore the concave ſpeculum has greatly the advantage of the ſphere and lenſes, for illuminating microſcopical objects, and alſo for burning things in the ſun-ſhine, though not in ſo great a proportion, as will appear by the next corollary. A compariſon of the burning powers of glaſſes and ſpeculums.

197. *Corol.* 5. Though the rays in two pictures of the ſun formed by ſimilar ſpecula, be equally denſe [a], yet the picture formed by the larger ſpeculum, being proportionably larger, will burn things more vehemently than the ſmaller; becauſe the burning particles of matter communicate and propagate their heat to one another. And when the ſpecula are ſimilar, the aberrations of the rays from the peripheries of the pictures, are alſo ſimilar [b].

[a] Art. 192.

[b] Art. 213.

CHAP. XI.

TO DETERMINE THE ABERRATIONS OF RAYS, FROM THE GEOMETRICAL FOCUS, CAUSED BY THEIR UNEQUAL REFRANGIBILITY, AND ALSO BY THE SPHERICALNESS OF THE FIGURE OF REFLECTING AND REFRACTING SURFACES.

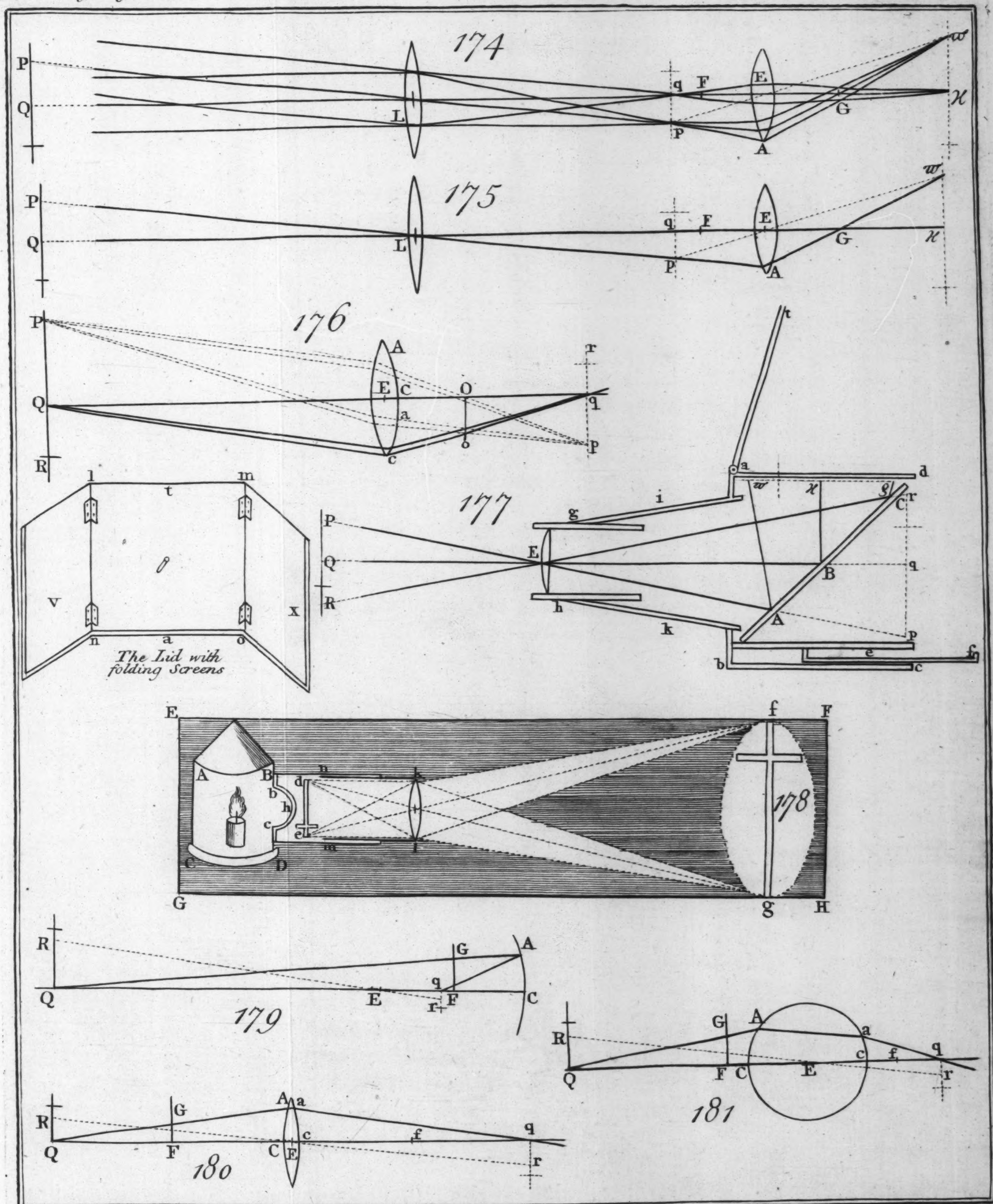
PROPOSITION I.

198. LET *the common ſine of incidence be to the ſine of refraction of the leaſt refrangible rays as* I *to* R, *and to the ſine of refraction of the moſt refrangible rays as* I *to* S; *and the diameter of the leaſt circular ſpace into which heterogeneal parallel rays can be collected, by a ſpherical ſurface or by a plano-convex lens, will be to the diameter of its aperture, in the conſtant ratio of* S — R *to* S + R — 2 I.

Fig. 182. For let an heterogeneal ray PA fall upon a ſpherical ſurface ACB, and let it be ſeparated by refraction into the rays AF, Af, cutting the axis EC, drawn parallel to PA, in F and f. Take the arch CB equal to CA, and let another heterogeneal ray PB, coming parallel to PA be refracted into the lines BF, Bf, cutting the two former rays in R and S. Join RS and produce it till it meets the incident rays produced in I and K, and the perpendiculars EA, EB to the refracting ſurface at the points A, B, in H and L. And when AB, the breadth of the aperture or of the pencil, is but moderate, and conſequently the refractions at A, B but ſmall, the angles of incidence and refraction, HAI, HAR, HAS, or the arches that meaſure them, or their perpendicular ſubtenſes HI, HR, HS, will be to each other very nearly in the ſame given ratios as thoſe of the ſines I, R, S of thoſe angles [a]. And disjointly the differences of thoſe ſubtenſes will be proportionable to the differences of theſe ſines: that is, the line $RS : RI :: S - R : R - I$, and doubling the conſequents, $RS : 2RI$ or $IK - RS :: S - R : 2R - 2I$; and conjointly $RS : IK$ or $AB :: S - R : S + R - 2I$. From this given ratio of RS to AB in which they increaſe or decreaſe together, it appears that all the intermediate rays which fall upon AB will paſs through RS. And when parallel rays fall perpendicularly upon the plane ſide of a plano-convex lens, they are refracted only at their emergence from its convex ſurface; and ſo the aberrations are the ſame in both caſes. *Q. E. D.*

[a] Art. 68. 84.

199. *Corol.*


174
175
176
177
178
179
180
181
The Lid with folding Screens

199. *Corol.* 1. Hence the diameter RS, of the circle of aberrations that contains all the incident rays, is a 55th part of the diameter AB of the aperture of a plano-convex glaſs, whatever be its focal diſtance. For ſuppoſing AR and AS to be the outmoſt red and indigo rays, their ſines of incidence and refractions I, R, S are to each other as 50, 77, 78[a]. Whence $S - R$ is to $S + R - 2I$ as 1 to 55. [a] Art. 31.

200. *Corol.* 2. The diameter of the leaſt circle that can receive the rays of any ſingle colour or of ſeveral contiguous colours is alſo determinable from the proportions of their ſines. Thus all the orange and yellow is contained in a circle whoſe breadth is the 260th part of the breadth of the aperture of the plano-convex glaſs; the ſines of the outermoſt orange AR and yellow AS being to the common ſine of incidence as $77\frac{1}{8}$ and $77\frac{1}{3}$ to 50[b]. [b] Art. 31.

201. *Corol.* 3. In different ſurfaces, or plano-convex glaſſes, the angles of aberration RAS are as the breadths of the apertures AB directly and as the focal diſtances CF inverſely; becauſe any angle, as RAS, is as its ſubtenſe RS directly and as its radius AR or CF inverſely.

202. *Corol.* 4. If ſeveral glaſſes of ſeveral ſorts or ſhapes have the ſame focal diſtance, and the ſame aperture; the diameter of the circle of aberrations of heterogeneal parallel rays from their principal focus, will be the ſame in them all; being the ſame as in a plano-convex glaſs when its plane ſide is turned to the incident rays[c]; and may therefore be determined by art. 198. And when the rays in the incident pencil are either parallel or inclined to the axis of the lens, the diameter of the circle of aberrations is as its diſtance from the lens; becauſe the angle RAS is invariable. [c] Art. 57. 58. Fig. 182.

203. *Corol.* 5. Therefore with reſpect to theſe aberrations by colours ſeparately conſidered, it is indifferent which ſide of the lens is turned to the incident rays; becauſe its focal diſtance is the ſame in both poſitions[d]. [d] Art. 101.

LEMMA.

204. *The verſed ſines* AB, AC *of very ſmall arches* BD, CD, *of unequal circles* BDG, CDH, *that have the ſame right ſine* AD, *are reciprocally proportionable to their diameters* BG, CH *very nearly*; *that is* AB : AC :: CH : BG. Fig. 183. 184.

For ſince the rectangles under BAG and CAH are each equal to the ſquare of AD[e], and conſequently to each other; their ſides are reciprocally proportionable[f], that is AB is to AC as AH to AG or [e] Euc. VI. 13. [f] Euc. VI. 14.

 as

as CH to BG very nearly, when the versed sines are incomparably
[a] Art. 68. less than the diameters themselves[a]. *Q. E. D.*

PROPOSITION II.

Fig. 185. 205. *When homogeneal parallel rays* NA, EC *fall upon a spherical surface* AC *whose center is* E, *the longitudinal aberration* FT *of any refracted ray* AT *from* F *the focus of the pencil, is to the versed sine of the arch* AC *intercepted between the point of incidence and the axis* ECF, *in the given ratio of the square of the sine of refraction, to the rectangle under the sine of incidence and the difference of the sines very nearly; and the aberration is the same when the rays fall perpendicularly upon the plane side of a plano-convex lens.*

For when the refraction is made in the passage of a ray NA from a denser to a rarer medium, the intersection T of the refracted ray AT with the axis ECF, lyes between the refracting surface and its focus F. With the center T and semidiameter TA having described the arch AD cutting the axis in D, draw the sine AP of the arches AC, AD, and also EN and EM the sines of incidence and refraction, for which put n and m; then because the triangles ETM, ATP are similar, it will be as $ET : TA$ or TD :: ($EM : AP$ or EN ::) $EF : FC$[b]; and disjointly $TF : EF$ :: ($FC - TD$ or) $TF - CD$: FC; and alternately $TF : TF - CD :: EF : FC$; and disjointly $TF : CD$ :: ($EF : EC$ ::) $m : m - n$[c]. Again since ($PD : PC$ :: $CE : DT$[d] or FC[e], and conjointly) $CD : CP$ :: ($EF : FC$ ::) $m : n$; by compounding this and the foregoing proportion, it will be as $TF : CP :: mm : \overline{m - n}, n$. *Q. E. D.*

[b] Art. 93.
[c] Art. 92.
[d] Art. 204.
[e] Art. 68.

206. *Corol.* 1. The segment $ACBPA$ may be considered as a plano-convex lens; and when rays fall parallel upon its plane side, the longitudinal aberration of the extreme ray falling upon A is equal to $\frac{9}{2}$ of its thickness PC, as appears by putting 3 and 2 for m and n respectively.

207. *Corol.* 2. Also this aberration $FT = \dfrac{mm}{\overline{m - n}, n} \times \dfrac{AP^2}{2EC} =$ $\dfrac{mm}{\overline{m - n}|^2} \times \dfrac{AP^2}{2CF}$. For $PC = \dfrac{AP^2}{2EC}$ very nearly[f], and $EC = \dfrac{m-n}{n}$ $\times CF$[g].

[f] Euc. VI. 13.
[g] Art. 92.

208. *Corol.* 3. Let the refracted ray ATG produced, cut the line FG, perpendicular to the axis, in G, and the lateral aberration

FG

$FG = \frac{mm}{nn} \times \frac{AP^3}{2EC^2} = \frac{mm}{\overline{m-n}|^2} \times \frac{AP^3}{2CF^2}$. For $FG : TF :: AP :$ TP or CF or $\frac{n}{m-n} \times CE$.

209. *Corol.* 4. When the ſemidiameter of the convexity or the focal diſtance is given, the longitudinal aberrations are as the ſquares, and the lateral aberrations as the cubes, of the linear apertures of a plano-convex lens.

PROPOSITION III.

210. *When parallel rays* QA, EC *are reflected from a ſpherical concave* ACB *whoſe center is* E *and whoſe aperture* ACB *is but ſmall, the longitudinal aberration* TF *of the extream ray* AT *from the geometrical focus* F, *is equal to half the verſed ſine* CP *of the ſemiaperture* AC *very nearly.* Fig. 186.

In fig. 185. imagine EM, the ſine of refraction to be diminiſhed to nothing, and then to become negative and equal to EN the ſine of incidence, and the refraction of the ray to be changed to reflection as in fig. 186; and by the former propoſition it will be as $TF : CP :: mm : -m-n, n :: nn : -2nn :: 1 : -2$.

But the particular proof is this. By the laſt lemma the verſed ſine CP nearly equals $\frac{1}{2}$ the verſed ſine PD of the arch AD whoſe center is T and ſemidiameter TA or TE or $\frac{1}{2}$ the ſemidiameter of the arch AC[a] very nearly. But $2TF = 2TE - 2EF = ED - EC$[a] $= CD$ exactly or CP nearly. Therefore $TF = \frac{1}{2}CP$ nearly. Fig. 186. Art. 69.

211. *Corol.* 1. We had $2TF = CD$ exactly; which is the exceſs of the ſecant ED of the arch AC above its radius EA. For joining AD the angle DAE in the ſemicircle DAE is a right one.

212. *Corol.* 2. The longitudinal aberration $TF = \frac{AP^2}{4CE}$. For $CP = \frac{AP^2}{2CE}$ nearly[b].

[b] Euc. VI. 13.

213. *Corol.* 3. The lateral aberration $FG = \frac{AP^3}{2CE^2}$. For $FG : FT :: AP : PT$ or $\frac{1}{2}CE$ nearly.

214. *Corol.* 4. When the diameter of the concave, or its focal diſtance, is given, the longitudinal aberrations are as the ſquares, and the lateral ones as the cubes of the diameters of the apertures.

PROPOSITION IV.

215. *When parallel rays of any one ſort are refracted by a plano-convex object-glaſs, or when rays of all ſorts are reflected by a ſpherical concave, the diameter of each circle of aberrations cauſed by the ſphericalneſs of the figures, is equal to* $\frac{1}{2}$ *the lateral aberration of the extream ray in each; and therefore is given by the former propoſitions.*

Fig. 187, 188. Let $\alpha\Upsilon\tau$ be any refracted or reflected ray cutting the axis ECT in τ, and the extream ray ATG, that comes from the contrary ſide of the axis, in Υ. Draw ΥX perpendicular to the axis, and ſuppoſing the line ATG immoveable, as the point of incidence α moves from the vertex C, the perpendicular $X\Upsilon$ will firſt increaſe, becauſe the angle $C\tau\alpha$ continually increaſes, and afterwards will decreaſe, becauſe the line $T\tau$ continually decreaſes; and when $X\Upsilon$ is the greateſt, it is evident that all the rays, incident upon the ſame ſide of the axis as itſelf, will paſs through it. To find its greateſt quantity, let the incident ray $q\alpha$ cut the chord APB in β, and ſuppoſing the variable aperture $P\beta = v$, the variable $TX = x$ and the given lines $PA = a$, $PT = f$, $TF = b$; by cor. 4. prop. 2 and 3, the aberration $F\tau$ is to the aberration FT (b) as $\varpi\alpha^2$ or $P\beta^2$ (vv) to PA^2 (aa). Wherefore $F\tau = \frac{vv}{aa}b$ and thence $TF - F\tau = T\tau = \frac{b}{aa}\times\overline{aa-vv}$. Again $PT\ (f) : PA\ (a) :: TX\ (x) : X\Upsilon = \frac{ax}{f}$; alſo $\varpi\alpha\ (v) : \varpi\tau$ or $PT\ (f) :: X\Upsilon\left(\frac{ax}{f}\right) : X\tau = \frac{ax}{v}$. Hence again $T\tau$, or $X\tau + XT = \frac{ax}{v} + x = \frac{b}{aa}\times\overline{aa-vv}$ found before; or $\frac{x}{v}\times\overline{a+v} = \frac{b}{aa}\times\overline{a+v}\times\overline{a-v}$. Whence $x = \frac{b}{aa}v \times\overline{a-v}$, and therefore x or TX is the greateſt poſſible when the rectangle $v\times\overline{a-v}$, or $P\beta\times\beta B$ is greateſt, that is when its ſides
a Euc. VI. 13. $P\beta$, βB are equal [a], or when $v=\frac{1}{2}a$. Subſtitute this value for v in the laſt equation and it gives the greateſt value of $x=\frac{1}{4}b$ or the greateſt $TX = \frac{1}{4}TF$, and therefore the greateſt $X\Upsilon = \frac{1}{4}FG$, becauſe $TX : X\Upsilon :: TF : FG$, and this $X\Upsilon$ turned about the axis PX deſcribes the circle of aberrations through which all the rays falling up AB will juſt paſs. Q. E. D.

PRO-

PROPOSITION V.

216. *If the density of the reflected rays in the circle of aberrations be uniform, it is to the density of the incident rays falling perpendicularly upon a plane* AP, *as the whole surface of the sphere of which the speculum is a portion, to the area of a circle whose diameter is the versed sine* PC *of the small arch* AC *very nearly, and the more exactly as this arch is smaller; supposing also that all the incident rays are reflected.* Fig. 188.

For since the very same rays pass through two circles described by the lines AP and XY turned about EC; their densities in these circles are reciprocally as the circles themselves; that is, the density of the reflected rays, is to the density of the incident rays, as AP^2 to XY^2 [a], or $\frac{1}{16}FG^2$ [b], or $\frac{AP^6}{16\times 4CE^4}$ [c]; that is, putting D for $2CE$, as $4D^4$ to AP^4; that is, as $4D^2$ to PC^2, (because D, AP, PC are very nearly continual proportionals [d],) that is, as the area of 4 great circles of the sphere, or the whole surface of the sphere [e], to the area of a circle whose diameter is PC very nearly.

[a] Euc. XII. 2.
[b] Art. 215.
[c] Art. 213.
[d] Euc. VI. 8. Cor.
[e] Archim. de Sph. & Cyl.

217. *Cor.* 1. Therefore the greatest density of the reflected rays is at the focus F, considered as a physical point; and is immensely greater than the density of the incident rays. For the proposition above becomes geometrically exact when AP is infinitely diminished, and XY comes to its limit at F; and the density at F is always the same whether a slender pencil falls upon the speculum or a large one, because the outward rays are reflected wide of the focus F.

218. *Corol.* 2. In like manner when rays fall parallel upon the plane side of a plano-convex lens, (putting m to n for the ratio of majority of the sines of incidence and refraction) their greatest density at their focus F, is to the density of the incident rays, as the whole surface of a sphere whereof the lens is a portion, to the area of a circle whose diameter is $\frac{mm}{nn}PC$, or in glass $\frac{9}{4}$ of the versed sine of the smallest aperture of the lens; that is immensely great. It follows from art. 208. Fig. 187.

219. *Cor.* 3. Therefore the density of reflected or refracted rays in the several points of an image of a very remote object, is also immensely greater than the density of the incident rays of any one pencil. For it would be immensely great, if all the rays of every pencil were rejected, except a few that go near to their axes, and those outward rays being scattered upon points collateral to each point

point of the image, help to increaſe the denſity of the rays in the whole image.

PROPOSITION VI.

220. *The circle of aberrations cauſed by the ſphericalneſs of the figure of the object-glaſs of a teleſcope, compared with the circle of aberrations cauſed by the unequal refrangibility of rays, is altogether inconſiderable.*

Newt. Opt. p. 83.

For if the object-glaſs be plano-convex and the plane ſide be turned towards the object, and the diameter of a ſphere whereof this glaſs is a ſegment be called D, and the ſemidiamer of the aperture of the glaſs be called S, and the ſine of incidence out of glaſs into air be to the ſine of refraction as n to m; the rays which come parallel to the axis of the glaſs ſhall in the place where the image of the object is moſt diſtinctly made, be ſcattered all over a little circle whoſe diameter is $\frac{mm}{nn} \times \frac{S^3}{DD}$ very nearly, if they were all equally refrangible by article 215 and 208. As for inſtance, if the ſine of incidence n be to the ſine of refraction m as 20 to 31, and if D, the diameter of the ſphere to which the convex ſide of the glaſs is ground, be 100 foot or 1200 inches, and conſequently the teleſcope about 100 foot long [a], and S the ſemidiameter of the aperture be 2 inches; the diameter of this circle of aberrations, that is $\frac{mm}{nn} \times \frac{S^3}{DD}$, will be $\frac{31 \times 31 \times 8}{20 \times 20 \times 1200 \times 1200}$ or $\frac{961}{72000000}$ parts of an inch. But the diameter of the little circle through which theſe rays are ſcattered by unequal refrangibility, will be about the 55th part of the breadth of the aperture of the object-glaſs [a], which is here 4 inches. And therefore the aberration ariſing from the ſpherical figure of the glaſs, is to the aberration ariſing from the different refrangibility, as $\frac{961}{72000000}$ to $\frac{4}{55}$, that is as 1 to 5449; and therefore being in compariſon ſo very little, deſerves not to be conſidered in the theory of teleſcopes. If we ſuppoſe the little circle of aberrations ariſing from unequal refrangibility, to be 250 times narrower than the circular aperture of the object-glaſs, it would contain all the orange and yellow, and would permit the other fainter and darker colours to paſs by it [c], which perhaps may ſcarce affect the ſenſe; yet even in this caſe the aberration cauſed by the ſpherical figure, would be to the aberration cauſed by the unequal refrangibility,

[a] Art. 92.

[b] Art. 199.

[c] Art. 200. Newt. Opt. p. 88.

lity, in a 100 foot telescope, but as $\frac{961}{72000000}$ to $\frac{4}{250}$, or only as 1 to 1200, which sufficiently proves the proposition. *Q. E. D.*

221. *Corol.* 1. If the focal distances and apertures of a reflecting concave and a plano-convex glass be both the same, the diameter of the circle of aberrations, caused by their figures, will be above 30 times less in the reflecter than in the refracter. For these diameters are $\frac{AP^3}{16CF^2}$ and $\frac{mm}{\overline{m-n}|^2} \times \frac{AP^3}{4CF^2}$ by art. 215, 213, and 208; which are as $\frac{1}{4}$ to $\frac{mm}{\overline{m-n}|^2}$ or $\frac{31 \times 31}{11 \times 11}$. Hence if the length of each telescope be 100 foot, the lateral aberrations in the reflecter would be 30×5449 or 163470 times less than the lateral aberrations caused by unequal refrangibility in the refracter.

222. *Corol.* 2. The number of pencils, some of whose rays are mixed together in every point of a confused picture, is as the area of the circle of aberrations of the rays in any one pencil; and consequently the mixture of the rays of different pencils, caused by the sphericalness of the figure of an object-glass, if they were all alike refrangible, would be to their mixture caused by their unequal refrangibility, as 1 to 5449×5449 or 29691601 in the present instance. For conceiving any point in the confused picture to be a center of a circle of aberrations, it is manifest that all other equal circles of aberrations, whose centers fall upon the first mentioned circle will cover its center; that is some rays of as many pencils will be mixed in this center as there are points in the circle itself; or, which is the same thing, the number of pencils mixed in this center is as the area of the circle of aberrations.

CHAP.

CHAP. XII.

A REFRACTING OR REFLECTING TELESCOPE BEING GIVEN, WHOSE APERTURE AND EYE-GLASS ARE ADJUSTED BY EXPERIENCE, TO DETERMINE THE LENGTH, APERTURE AND EYE-GLASS OF ANOTHER TELESCOPE, THROUGH WHICH AN OBJECT SHALL APPEAR AS BRIGHT AND DISTINCT AS IN THE GIVEN ONE, AND MAGNIFIED AS MUCH AS SHALL BE REQUIRED.

PROPOSITION I.

223. IN *all sorts of telescopes and double microscopes, the apparent indistinctness of a given object, is as the area of a circle of aberrations in the focus of the object-glass directly, and as the square of the focal distance of the eye-glass inversely.*

For in vision with the naked eye or with glasses, the apparent indistinctness of a given object, is as the area of a circle of aberrations in its picture painted upon the retina. Because any one sensible point of the retina, being the center of a circle of aberrations, will at once be affected by a mixture of the rays of as many distinct pencils, as there are sensible points in the area of that circle [a]; and so will at once convey to the mind a mixt or confused sensation of the same number of visible points in the object, from whence those pencils flowed; and this number of points is as the magnitude of the area of a circle of aberrations, whatever be the magnitude of a sensible point of the retina. Now in vision with telescopes, the diameter of a circle of aberrations in the picture upon the retina, is as the apparent magnitude of the diameter of the corresponding circle of aberrations in the common focus of the glasses [b], that is as the angle subtended by this diameter at the center of the eye-glass [c]; that is as the diameter itself directly, and the focal distance of the eye-glass inversely [d]. And so the area of that circle of aberrations upon the retina, is as the area of the corresponding circle of aberrations in the focus of the object-glass directly, and as the square of the focal distance of the eye-glass inversely [e].

[a] Art. 222.
[b] Art. 136.
[c] Art. 164.
[d] Art. 86.
[e] Euc. XII. 2.

224. *Corol.* In all sorts of telescopes and double microscopes a given object appears equally distinct, when the focal distances of the eye-

eye-glaſſes are as the diameters of the circles of aberrations in the focus of the object-glaſſes.

225. The alteration in the confuſion which may ariſe from aberrations cauſed by the eye-glaſſes, is not here regarded, as being inconſiderable. We only conſider the confuſion of thoſe points in the image which lye very near the axis of the teleſcope, as of the point q in fig. 162. Now if this point was perfectly diſtinct the rays going from it would emerge from the eye-glaſs in parallel lines without ſenſible error; becauſe the breadth of this cylinder of rays is exceeding ſmall compared to the breadth of the eye-glaſs, being in proportion to the breadth of the aperture of the object-glaſs as their focal diſtances; and the refractions at ſo ſmall a diſtance from the axis are ſufficiently true and regular. It is the largeneſs of the aperture of the object-glaſs and of its focal diſtance, which cauſes the irregularity in its refractions. Add to this that the differently refrangible rays cannot be ſeparated ſenſibly in going ſo ſhort a diſtance as between the eye-glaſs and the eye. Beſides this we find by experience that objects and images diſtinct in themſelves, appear ſufficiently diſtinct through very ſmall eye-glaſſes when their apertures are ſmall.

PROPOSITION II.

226. *In refracting teleſcopes the apparent indiſtinctneſs of a given object, is directly as the area of the aperture of the object-glaſs, and inverſely as the ſquare of the focal diſtance of the eye-glaſs.*

This appears from prop. 1, becauſe the area of the circle of aberrations at the focus of the object-glaſs is as the area of its aperture[a]; and becauſe the aberrations ariſing from the eye-glaſs[b], and from the ſphericalneſs of the figure of them both are inconſiderable[c]. [a] Art. 198. [b] Art. 225. [c] Art. 220.

227. *Corol.* In refracting teleſcopes a given object appears equally diſtinct, when the diameters of the apertures of their object-glaſſes, are as the focal diſtances of their eye-glaſſes.

PROPOSITION III.

228. *In all ſorts of teleſcopes and double microſcopes the apparent brightneſs of a given object is as the ſquare of their linear apertures directly and as the ſquare of their linear amplifications inverſely.*

For if the ſquares of the linear amplifications, that is if the areas of the pictures upon the retina were the ſame, their brightneſs

would be as the quantities of light coming through the areas of the apertures, that is as the ſquares of the linear apertures; and if the apertures or quantities of light were the ſame, the brightneſs of the pictures would be as their areas inverſely or as the ſquares of the linear amplifications inverſely. Therefore when neither the apertures nor the amplifications are the ſame, the brightneſs is as the ſquare of the linear apertures directly, and as the ſquare of the linear amplifications inverſely. *Q. E. D.*

229. *Corol.* 1. Hence in refracting and reflecting teleſcopes a given object appears equally bright, when their linear apertures are as their linear amplifications, that is as the focal diſtances of the object-glaſſes directly and as the focal diſtances of the eye-glaſſes inverſely.

230. *Corol.* 2. If the breadth of the aperture of a given object-glaſs and the focal diſtance of the eye-glaſs be each increaſed in any given ratio, the diſtinctneſs will remain the ſame as before [a]; and the linear amplification will be diminiſhed in the ſame ratio [b]; but the apparent brightneſs will be increaſed in a ratio quadruplicate of the former ratio by this propoſition; and on the contrary.

[a] Art. 227.
[b] Art. 164.

231. *Hugens* obſerves that the ſame degrees of diſtinctneſs here demonſtrated do not exactly agree with experience, as he found by looking at the ſame object through different teleſcopes, or through the ſame teleſcope with different apertures; and that through the larger aperture the object appeared not quite ſo diſtinct as through the ſmaller. He found alſo that in viewing objects of different brightneſs through the ſame aperture, the apparent indiſtinctneſs of the brighter object was a little greater than that of the duller: and therefore the aperture adjuſted for the duller planets may be ſomewhat larger than for the brighter.

Dioptr. p. 215.

PROPOSITION IV.

232. *In reflecting teleſcopes the apparent indiſtinctneſs of a given object is as the ſixth power of the diameter of the aperture of the object-metal directly, and as the fourth power of its focal diſtance inverſely, and alſo as the ſquare of the focal diſtance of the eye-glaſs inverſely.*

For the area of a circle of aberrations in the focus of the object-metal is as the ſixth power of its linear aperture directly and as the fourth power of its focal diſtance inverſely [c]; and therefore the apparent indiſtinctneſs of the object, is as the ſixth power of the linear aperture

[c] Art. 215. 213.

aperture directly, as the fourth power of the focal diſtance of the object-metal inverſely, and as the ſquare of the focal diſtance of the eye-glaſs inverſely[a]. *Q. E. D.* [a] Art. 223.

233. *Corol.* In reflecting teleſcopes a given object appears equally diſtinct when the cubes of the linear apertures of the object-metals, are as the ſolids whoſe baſes are the ſquares of the focal diſtances of the object-metals, and heights are the focal diſtances of the eye-glaſſes: or when the focal diſtances of the eye-glaſſes are as the cubes of the linear apertures of the object-metals, applied to the ſquares of their focal diſtances.

PROPOSITION V.

234. *In refracting teleſcopes of various lengths a given object will appear equally bright and equally diſtinct, when their linear apertures and focal diſtances of their eye-glaſſes are ſeverally in a ſubduplicate ratio of their lengths or focal diſtances of their object-glaſſes: and then alſo their linear amplifications will be in a ſubduplicate ratio of their lengths.*

For to ſhew the object equally bright, the rectangle under the linear aperture and the focal diſtance of the eye-glaſs muſt be as the length of the teleſcope[b], and to ſhew it equally diſtinct the linear [b] Art. 229.
aperture muſt be as the focal diſtance of the eye-glaſs[c]; and there- [c] Art. 227.
fore to perform both things together, the ſquare of the linear aperture, and alſo the ſquare of the focal diſtance of the eye-glaſs, muſt be ſeverally (as the rectangle under each, or) as the length of the teleſcope; and conſequently the linear aperture, and alſo the focal diſtance of the eye-glaſs, as the ſquare root of that length. Now the linear amplification was as the linear aperture[d], or by this demon- [d] Art. 227.
ſtration, as the ſquare root of the length of the teleſcope. *Q. E. D.*

235. *Hugens*'s ſtandard teleſcope 30 foot long, or 360 inches, Diop. p. 210.
bears an aperture whoſe breadth is 3 inches, and an eye-glaſs whoſe focal diſtance is 3 inches and 3 tenths. From whence he has given us the following table of apertures and eye-glaſſes for other teleſcopes[e], computed by the following rule. [e] Art. 244.

Multiply the number of feet in the focal diſtance of any propoſed object-glaſs by 3000, and the ſquare root of the product will give the breadth of its aperture in hundredth parts of an inch. And the ſame breadth of the aperture, increaſed by a tenth part of itſelf, gives the focal diſtance of the eye-glaſs in hundredth parts of an inch. And the magnifying powers are as the breadths of the apertures.

For ſince the ſtandard teleſcope has 30 foot focal diſtance of its

object-glaſs, put F for the number of feet in any other focal diſtance, and ſay by the propoſition as $\sqrt{30}$ to $\sqrt{F}$, ſo is the ſtandard aperture 3 inches or 300 centeſimals or $\sqrt{300 \times 300}$, to the aperture ſought; which therefore is $\sqrt{3000F}$ in centeſimals of an inch. The focal diſtance of the eye-glaſs of the ſtandard teleſcope is $3\frac{3}{10}$ inches, that is a tenth part more than the breadth of the aperture of the object-glaſs; conſequently the focal diſtance of the new eye-glaſs muſt be a tenth part more than the linear aperture of the new object-glaſs, by the laſt propoſition.

Diop. p. 215. 236. He alſo adds the following directions how to ſuit theſe teleſcopes to all ſorts of objects ſeen either by day or by night. They are proportioned in the following table for aſtronomical obſervations, and therefore will require more light when uſed in the day time. For when the eye is dazled with the brightneſs of the day, objects will appear through them but obſcure, which in the night are ſufficiently bright. Therefore (ſays *Hugens*) when I uſed theſe teleſcopes to obſerve objects by day-light, by experience I found it requiſite to change the eye-glaſſes for others whoſe focal diſtances were double the former. By this means the apparent brightneſs became quadruple, becauſe the ſurfaces of the images in the bottom of the eye [a] Art. 164. were diminiſhed in the ſame proportion [a]. For as the aperture remains unaltered, ſo does the quantity of light, and therefore it illuminates a leſſer ſpace ſo much the more. Now if the aperture was increaſed without changing the eye-glaſs, the brightneſs would be increaſed too, but then the miſt ariſing from greater aberrations would alſo be greater; and therefore this remedy muſt not be uſed.

237. But one may aſk this queſtion, ſince by ſubſtituting an eye-glaſs of a longer focal diſtance, the apparent indiſtinctneſs hitherto examined is diminiſhed, why may not the aperture of the object-glaſs be ſo far increaſed, till the ſame degree of indiſtinctneſs returns again as belongs to a teleſcope regulated by the table? For from [b] Art. 230. hence more light is gained and the diſtinctneſs is not altered [b]. The [c] Art. 231. anſwer is this, which I hinted before [c], that the miſt ariſing from *Newton*'s aberration, though the ſame in quantity, becomes more ſenſible in proportion to the brightneſs of the image. For the brightneſs of the miſt increaſes at the ſame time. And we find by experience, that as ſoon as the apertures of thoſe day-light teleſcopes are increaſed, the miſt ariſing from the aberrations of a brighter object begins to be troubleſome. The apertures therefore muſt not be altered.

238. Again one may aſk, if a teleſcope fitted for Saturn be applied to the Moon, which is 100 times brighter (I mean in each equal

parts

parts, though not in the whole, as being 10 times nearer to the Sun;) one may aſk I ſay whether the breadth of the aperture and the focal diſtance of the eye-glaſs may not both be leſſened in the ſame proportion to make the regions of the moon no brighter than thoſe of ſaturn, but much greater in appearance than before. For inſtance, in a 30 foot teleſcope, if 3 inches, the breadth of the aperture be reduced to $\sqrt{\frac{9}{10}}$ of an inch, which is ſomewhat leſs than ($\sqrt{\frac{9}{9}}$ or) a third part of the former, and alſo the focal diſtance of the eye-glaſs be ſhortened in the ſame proportion; the proportion of the apparent brightneſs in theſe two teleſcopes, the object being the ſame, would be quadruplicate of 3 to $\sqrt{\frac{9}{10}}$ [a] that is as 100 to 1; [a] Art. 230.
and ſince the regions in the moon are 100 times brighter in themſelves than thoſe in ſaturn, the moon would appear in the darker teleſcope juſt as bright as ſaturn did in the lighter. But the apparent indiſtinctneſs hitherto conſidered would alſo be the ſame in both [b], and the amplification of the moon would be greater than [b] Art. 230.
that of ſaturn in the ratio of 3 to $\sqrt{\frac{9}{10}}$ [c], which is more than triple. [c] Art. 164.
So that this reduction of the aperture and eye-glaſs ſeems very advantageous; but in reality it is quite otherwiſe; and that for two reaſons. Firſt becauſe the minute parts of the moon may be better diſcerned when all the light remains in the teleſcope, than when it is reduced to an 100th part, though not in the ſame proportion. The other reaſon is that when the aperture is too much contracted, the out-lines that circumſcribe the pictures in the eye become confuſed; which is carefully to be minded, and alſo what are the limits of this confuſion. This is certain that as the aperture is contracted, the ſlender pencils or cylinders of rays, that emerge from the eye-glaſs into the eye, are alſo contracted in the ſame proportion. Now if the breadth of one of theſe pencils be leſs than $\frac{1}{5}$ or $\frac{1}{6}$ of a line, that is leſs than $\frac{1}{60}$ or $\frac{1}{72}$ part of an inch, the out-lines of the pictures are ſpoiled, for ſome unknown reaſon in the make of the eye, whether in the choroid, or in the retina, or in the humors it is uncertain. For by looking through an hole, in a thin plate, narrower than $\frac{1}{5}$ or $\frac{1}{6}$ of a line, the edges of objects begin to appear confuſed and ſo much the more as the hole is made narrower. Now it is eaſy to ſhew in the laſt mentioned teleſcope that the cylinder of rays is too ſlender. For by adding $\frac{1}{10}$ of the aperture to itſelf [d], the [d] Art. 235.
focal diſtance of the eye-glaſs becomes $\sqrt{\frac{9}{10}} + \frac{1}{10}\sqrt{\frac{9}{10}}$, that is $\frac{11}{10}\sqrt{\frac{9}{10}}$ of an inch; and by ſimilar triangles ſubtended at the common focus q by the aperture and cylinder ſought [e], it is as the focal di- [e] Fig. 162.
ſtance of the object-glaſs, to the focal diſtance of the eye-glaſs, ſo the breadth of the aperture, to the breadth of the cylinder; that is as

as 30 feet or 360 inches to $\frac{11}{10}\sqrt{\frac{9}{10}}$ inches, ſo is $\sqrt{\frac{9}{10}}$ of an inch to $\frac{11}{4000}$ inch or almoſt $\frac{1}{30}$ of a line; which is much leſs than $\frac{1}{6}$. But in the teleſcope regulated in the table, it is as 360 to $3\frac{1}{10}$ ſo 3 to $\frac{11}{400}$ of an inch or almoſt $\frac{1}{3}$ of a line for the breadth of that cylinder; which can poſſibly do no harm. Hence we learn that the breadth of the aperture and focal diſtance of the eye-glaſs cannot be contracted much more than $\frac{1}{3}$ of themſelves; for even then the breadth of the cylinder at the eye will not much exceed $\frac{1}{5}$ of a line. The ſame is to be underſtood of teleſcopes of all lengths regulated as in the table, the breadth of the cylinder being the ſame in all. For by the proportion juſt mentioned it equals the breadth of the aperture multiplied into the focal diſtance of the eye-glaſs and divided by the focal diſtance of the object-glaſs, and conſequently it is proportionable to the linear aperture directly and the linear amplification inverſely; which two ratios muſt compound a ratio of equality to preſerve the ſame apparent brightneſs, by art. 229.

239. Hence though we transferred one of theſe teleſcopes from Saturn to Venus which is 225 times brighter, being 15 times nearer to the Sun, yet the breadth of the aperture muſt not be contracted above $\frac{1}{3}$ part of the whole; and if too much light ſtill remains, it muſt be diminiſhed by darkening the eye-glaſs with the ſmoak of a candle. For a greater contraction of the aperture is hurtful for another reaſon, that all the little bubbles and veins in the eye-glaſs become more conſpicuous by intercepting the whole or a greater part of thoſe little cylinders above mentioned, and conſequently the particles of the object they came from.

240. Upon the whole I conclude we may lengthen our teleſcopes at pleaſure, according to the laws of the table, with good ſucceſs; ſince not only the brightneſs and diſtinctneſs remain unaltered, but alſo the breadth of the pencils that enter the eye. Laſtly to obſerve exceeding ſmall ſtars and eſpecially the Satellites of Jupiter and Saturn, the beſt way is to increaſe very much both the aperture and focal diſtance of the eye-glaſs. For ſince they appear like points even through the teleſcopes, there is nothing gained by endeavouring to increaſe their diameters; but their brightneſs muſt be increaſed as much as poſſible; and this is chiefly done by increaſing the aperture. By doubling its breadth, the light received into it becomes quadruple, and then by doubling alſo the focal diſtance of the eye-glaſs, the diſtinctneſs returns to the ſame as at firſt [a]. But ſtill the brightneſs will not become 16 times greater, according to cor. 2. prop. 3, but only 4 times; becauſe as I ſaid the picture of the ſtar upon the retina is but a ſenſible point, whoſe brightneſs cannot

[a] Art. 227.

cannot therefore be increaſed by a diminution of its breadth, but only by an addition of new light. The caſe is different when we view the moon and primary planets through the ſame teleſcope, whoſe ſeveral parts receive 16 times more light than before. Thus by widening the apertures we very much increaſe the power of the teleſcope for finding out ſmall ſtars and the ſattellites of Saturn, ſo that perhaps with a 30 foot glaſs, whoſe aperture is 6 inches or double the uſual one, as much may be done as with another of 120 foot whoſe aperture by the table is alſo 6 inches. So far from *Hugens*.

PROPOSITION VI.

241. *In reflecting teleſcopes of various lengths a given object will appear equally bright and equally diſtinct, when their linear apertures and alſo their linear amplifications are as the ſquare-ſquare roots of the cubes of their lengths: and conſequently when the focal diſtances of their eye-glaſſes are alſo as the ſquare-ſquare roots of their lengths.*

Put A for the linear aperture of the reflecting concave, L for its focal diſtance or the length of the teleſcope, F for the focal diſtance of the eye-glaſs; and when the diſtinctneſs is given A^3 is as FLL[a]; [a] Art. 233. and when the brightneſs is given the amplification or $\frac{L}{F}$ is as A[b], [b] Art. 229. that is F is as $\frac{L}{A}$. Therefore when the diſtinctneſs and brightneſs are both given, A^3 is as $\frac{L^3}{A}$; or A^4 as L^3; or A as $\sqrt[4]{L^3}$. The amplification $\frac{L}{F}$ was as A, that is as $\sqrt[4]{L^3}$; and therefore F is as $\frac{\sqrt[4]{L^4}}{\sqrt[4]{L^3}}$ or $\sqrt[4]{L}$, Q. E. D.

242. In the reflecting teleſcope made and deſcribed by *John Hadley*, Eſq; F. R. S. in the Philoſophical Tranſactions N°. 376 and 378, $L = 62\frac{1}{2}$ inches, $F = \frac{1}{3}$ or $\frac{3}{10}$ or $\frac{11}{40}$ of an inch. For he uſes 3 eye-glaſſes and as many apertures for the reflecter whoſe breadths are $4\frac{1}{2}$, 5, $5\frac{1}{2}$ inches. Hence the linear amplifications or $\frac{L}{F}$ are $187\frac{1}{2}$, $208\frac{1}{3}$, $227\frac{3}{11}$ reſpectively. Taking the middle eye-glaſs and aperture for a ſtandard I computed the following table for teleſcopes of other lengths by this Rule. Call the number of inches in the length of

of any telescope L, and the focal distance of its eye-glass will be equal to $60\sqrt[4]{10L}$ in thousandth parts of an inch. The quotient of L divided by $60\sqrt[4]{10L}$ or F gives the amplification [a], which multiplied by 24 will always give the linear aperture in thousandth parts of an inch. For by the proposition $\sqrt[4]{L}$ is as F; that is $\sqrt[4]{62\frac{1}{2}}$ or $\sqrt[4]{\frac{125}{2}}$ or $\sqrt[4]{\frac{625}{10}}$ or $5\sqrt[4]{\frac{1}{10}}$ is to $\sqrt[4]{L}$ as $\frac{3}{10}$ or 300 millesimals in the given eye-glass, to the millesimals in the correspondent eye-glass or in $F = 60\sqrt[4]{10L}$. And the aperture being as the amplification by the proposition, say, as the amplification given or $208\frac{1}{3}$ is to $\frac{L}{F}$, the amplification found, so is 5 inches, the aperture given, to the aperture sought $= \frac{5}{208\frac{1}{3}} \times \frac{L}{F} = \frac{24}{1000} \times \frac{L}{F}$ inches.

[a] Art. 169.

243. Were it not for the unequal refrangibility of rays, refracting telescopes, though not so short as these [b], would also be proportioned by this rule [c]: which not agreeing with experience, shews again that the aberrations arising from the spherical figure are inconsiderable in comparison to the other aberrations arising from the unequal refrangibility of the rays.

[b] Art. 221.

[c] Art. 209. 214.

244. REFRACT-

244. REFRACTING TELESCOPES

Length of the telescope or focal dist. of the object-glass. *Feet.*	Linear aperture of the object-glass. *Inch.&Dec*	Focal dist. of the eye-glass. *Inch &Dec.*	Linear amplification, or magnifying power.
1	0. 55	0. 61	20
2	0. 77	0. 85	28
3	0. 95	1. 05	34
4	1. 09	1. 20	40
5	1. 23	1. 35	44
6	1. 34	1. 47	49
7	1. 45	1. 60	53
8	1. 55	1. 71	56
9	1. 64	1. 80	60
10	1. 73	1. 90	63
13	1. 97	2. 17	72
15	2. 12	2. 32	77
20	2. 45	2. 70	89
25	2. 74	3. 01	100
30	3. 00	3. 30	109
35	3. 24	3. 56	118
40	3. 46	3. 81	126
45	3. 67	4. 04	133
50	3. 87	4. 26	141
55	4. 06	4. 47	148
60	4. 24	4. 66	154
70	4. 58	5. 04	166
80	4. 90	5. 39	178
90	5. 20	5. 72	189
100	5. 48	6. 03	199
120	6. 00	6. 60	218
140	6. 48	7. 13	235
160	6. 93	7. 62	252
180	7. 35	8. 09	267
200	7. 75	8. 53	281
220	8. 12	8. 93	295
240	8. 48	8. 83	308
260	8. 83	9. 71	321
280	9. 16	10. 08	333
300	9. 49	10. 44	345
400	10. 95	12. 05	398
500	12. 25	13. 47	445
600	13. 42	14. 76	488

REFLECTING TELESCOPES

Length of the telescope or focal dist. of theconcave *Feet.*	Focal dist. of the eye-glass. *Millef. Inch.*	Linear amplification, or magnifying power.	Linear aperture of the concave-metal. *Millef. Inch.*
$\frac{1}{2}$	0. 167	36	0. 864
1	0. 199	60	1. 440
2	0. 230	102	2. 448
3	0. 261	138	3. 312
4	0. 281	171	4. 104
5	0. 297	202	4. 848
6	0. 311	232	5. 568
7	0. 323	260	6. 240
8	0. 334	287	6. 888
9	0. 344	314	7. 536
10	0. 353	340	8. 160
11	0. 362	365	8. 760
12	0. 367	390	9. 360
13	0. 377	414	9. 936
14	0. 384	437	10. 488
15	0. 391	460	11. 040
16	0. 397	483	11. 592
17	0. 403	506	12. 143

245. These propositions, in *Hugens*'s table for refracting telescopes, are measured by the Rheinland foot which is to the English foot as 139 to 135; so that taking their lengths of as many English feet, their apertures and eye-glasses and linear amplifications should be severally diminished in the subduplicate ratio of 139 to 135 by art. 234. that is nearly in the ratio of 139 to 137 or about $\frac{1}{68}$ or $\frac{1}{70}$ part of the whole.

CHAP. XIII.

CONCERNING THE RAIN-BOW.

LEMMA I.

Fig. 189. 246. THE *ratio of the tangents*, CT, CV, *of any two angles*, CBD, CBE, *is compounded of the ratio of their sines*, CD, CE, *taken directly, and of their cosines*, BD, BE, *taken inversely*.

For the right angled triangles *BCT*, *BDC* are equiangular, and so are the right angled triangles *BCV*, *BEC*. Therefore the ratio of *CT* to *CV*, which is compounded of *CT* to *CB* and of *CB* to *CV*, or of *CD* to *DB* and of *EB* to *EC*, is the same as the ratio of the rectangle under *CD*, *EB* to the rectangle under *DB*, *EC*, which is compounded of the ratio of *CD* to *CE* and of *EB* to *DB* [a], that is of the sines directly and cosines inversely. *Q. E. D.*

[a] Euc. VI. 23.

LEMMA II.

Fig. 190, 191. 247. *The least increment of an angle of incidence, is to the contemporary increment of the angle of refraction, as the tangent of the angle of incidence, to the tangent of the angle of refraction.*

Let two rays *AB*, *aB*, containing a very small angle *ABa*, be refracted at *B* along the lines *BE*, *Be* by a plane or by any curve-surface. From any point *C*, of the line *BC* perpendicular to that surface, draw *CDd* cutting the incident rays (produced) at right angles [b] in *D* and *d*; and likewise *CEe* cutting the refracted rays (produced) at right angles in *E* and *e*. Then because *CD* is to *CE* and *Cd* to *Ce* in the same ratio of the sines, disjointly we have *Dd* to *Ee* as *CD* to *CE*. Now the ratio of the small angles (*ABa* or) *DBd* and *EBe*, which are the contemporary increments or decrements of the angles of incidence and refraction, being compounded of the ratio of *Dd* to *Ee* and of *BE* to *BD* [c], that is of *CD* to *CE* and of *BE* to *BD*, is the same as the ratio of the tangents of incidence and refraction [d]. *Q. E. D.*

[b] Art. 68.
[c] Art. 86.
[d] Art. 246.

PRO-

PROPOSITION I.

248. *When a ray of light is refracted into a circle, and successively reflected within it any given number of times before it emerges out of the circle by a second refraction; let the angle of refraction be multiplied by the number of successive reflections increased by an unite; and the excess of the resulting angle above the angle of incidence will be equal to half the angle contained under the incident and the emergent ray produced till they meet: that is, the excess abovementioned is equal to half that angle, under the incident and the emergent ray, in which the refracting circle lyes, when the number of reflections is odd; and is equal to half the other angle, under the same rays, which is the complement of the former to two right angles, when the number of reflections is even.*

For let $ABCDE$ be a great circle of a sphere whose center is O, Fig. 192. to 195.
and let an incident ray SA be refracted at A to B, and be reflected from B to C; and at C let it either go out by refraction to G, or be reflected to D[a]; where let it either go out by refraction to H, or be reflected to E; and so on. And when the number of reflections is odd, a line OR drawn through the center O and the middlemost point of reflection, will bisect the angle at R under the incident and the emergent ray produced: because the reflections and refractions on each side of the line OR are equal in number and magnitude; the chords AB, BC, CD, DE described by the reflected ray being equal to one another. And for the same reason when the number of reflections is even, a line OT, drawn through the center O perpendicular to the chord that joins the two middlemost points of reflection, will bisect one of the angles at T under the incident and the emergent ray produced; and a line TV, perpendicular to TO, will bisect the other angle under them, which is the complement of the former to two right ones. Hence the line TV is parallel to the middlemost chord, because TO is perpendicular to them both. Draw a diameter POQ parallel to the incident ray SAM, and let it cut the reflected rays BC, CD, DE produced, in β, γ, δ, respectively. Join OA, OB and in fig. 192. the sums of the three angles in each of the triangles OAB, OAR, are equal to one another; take away the common angle AOB, and the sum of the equal angles OAB, OBA in the first triangle, will be equal to the sum of the angles OAR, ORA in the second triangle. And by substracting the angle of incidence OAR or OAM from both sums, we have $2\,OAB - OAM = ORA = BOQ$. Hence in fig. 193. the angle STV or $P\beta C$, being an external angle of the triangle $OB\beta$, equals $OBC +$

[a] Art. 35, &c.

 BOQ

$BOQ = OAB + 2OAB - OAM = 3OAB - OAM$. Hence again in fig. 194. the angle SRO or POC, being an external angle of the triangle $OC\beta$, equals $OCB + P\beta C = OAB + 3OAB - OAM = 4OAB - OAM$. Hence again in fig. 195. the angle STV or $P\gamma D$, being an internal angle of the triangle $CO\gamma$, equals $OCD - CO\gamma = 5OAB - OAM$, throwing away two right angles. For $CO\gamma = 2$ right angles $- POC = 2$ right angles $- 4OAB + OAM$. And ſo forward continually. Therefore if the number of ſucceſſive reflections increaſed by an unite be called m, it appears that $mOAB - OAM$ equals half the angle under the incident and emergent rays. *Q. E. D.*

PROPOSITION II.

249. *Things remaining as they were, let the angle of incidence increaſe from nothing till it becomes a right angle; and the angle under the incident and the emergent ray, after any given number of reflections called* n, *will firſt increaſe and then decreaſe again; and will be the greateſt of all when the tangent of the angle of incidence, is to the tangent of the angle of refraction, as* n + 1 *to* 1.

Fig. 192. to 195. For putting $m = n + 1$, we had half the angle under an incident and the emergent ray equal to the exceſs of $mOAB$ above OAM[a]; which exceſs, when the angles OAB, OAM are very ſmall, will alſo be but ſmall; and will increaſe ſo long as the ſucceſſive increments of $mOAB$ ſhall exceed the contemporary increments of OAM; and will decreaſe again when the ſucceſſive increments of $mOAB$ are exceeded by the increments of OAM; and conſequently will be the greateſt of all when m times the leaſt increment of OAB is equal to once the contemporary increment of OAM; that is when the leaſt increment of the angle of incidence OAM is to the contemporary increment of the angle of refraction OAB, and conſequently the tangent of incidence is to the tangent of refraction[b], as m to 1. *Q. E. D.*

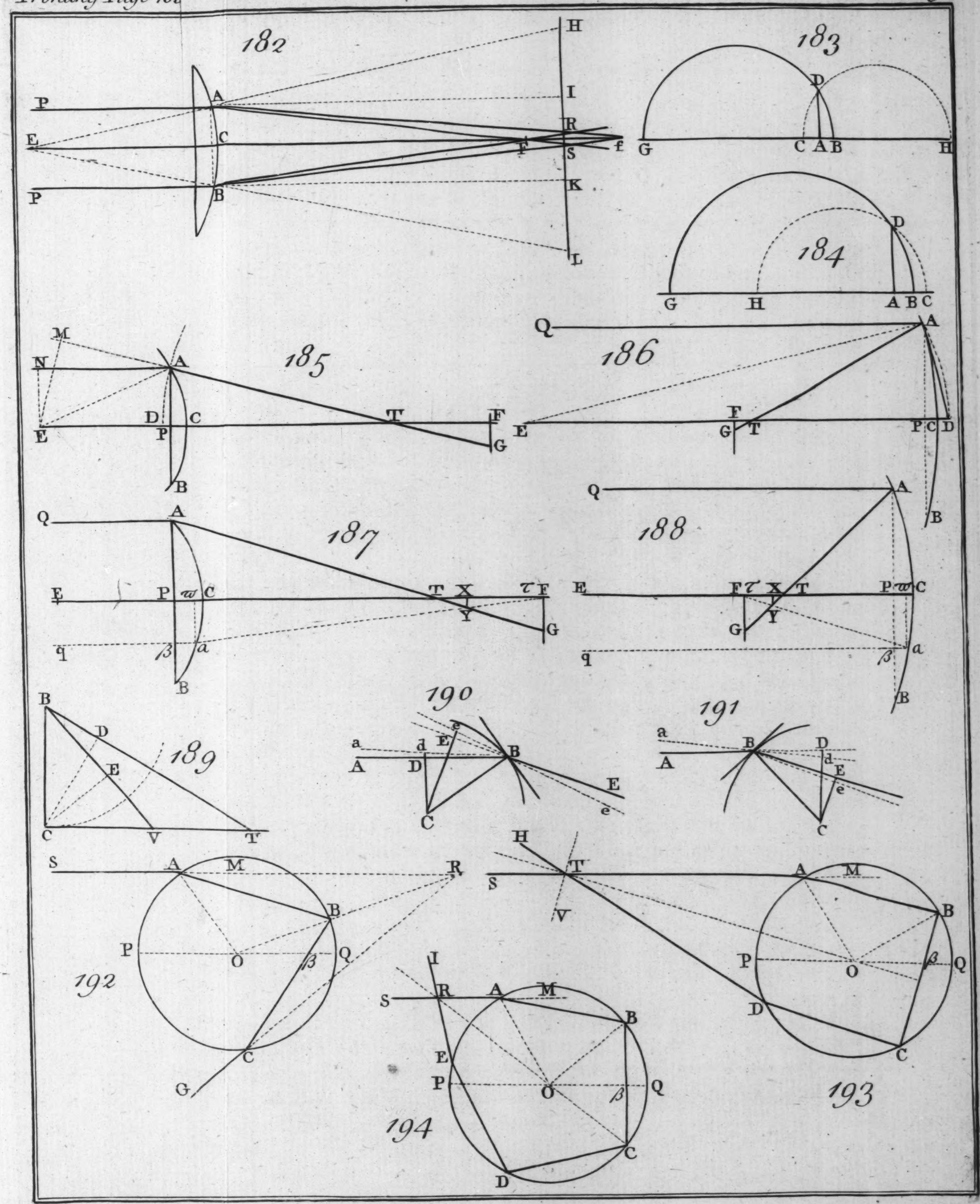
[a] Art. 248.

[b] Art. 247.

PROPOSITION III.

250. *It is propoſed to find two angles, whoſe ſines ſhall be in a given ratio of* I *to* R, *and whoſe tangents ſhall be in another given ratio of* m *to* 1.

Fig. 196. In any given line $CEDA$, let CA be to CD as I to R, and CA to CE as m to 1; with the center C and ſemidiameter CD deſcribe an arch DB, cutting a circle ABE whoſe diameter is AE, in B; draw ABF, and joining BC, the ſine of the angle CBF will be to the ſine


182
183
184
185
186
187
188
189
190
191
192
193
194

ſine of CAF as I to R; and the tangent of CBF to the tangent of CAF as m to 1; and conſequently CBF, CAF are the angles required. For in the triangle CAB the ſine of the angle CBA or CBF, is to the ſine of CAF, as CA to CB[a] or CD, as I to R by conſtruction. Join BE and compleat the parallelogram $CEBG$; and CG produced will cut ABF at right angles in F, becauſe ABE is a right angle in the ſemicircle ABE. Therefore the lines FC, FG are tangents of the angles CBF, GBF or CAF to the radius BF; and the tangent FC is to the tangent FG as FA to FB[b] or as CA to CE[b] or as m to 1 by conſtruction. *Q. E. D.*

[a] Art. 85.

[b] Euc. VI. 2.

251. *Corol.* 1. When parallel rays of the ſun fall upon a ſpherical drop of rain, let the given ratio of I to R ſtand for the ratio of the ſine of incidence to the ſine of refraction; and let n be any given number of ſucceſſive reflections made by every ray before it emerges out of the drop, and let $m = n + 1$; and by theſe propoſitions it appears, that half the greateſt angle which any of the emergent rays can make with the incident rays, is equal to $m \times$ ang. $CBF - CAF$. For CBF and CAF or GBF are angles whoſe ſines are as I to R, and whoſe tangents are as m to 1; and conſequently are the angles of incidence and refraction of that ray, whoſe incident and emergent parts produced contain the greateſt angle.

252. *Corol.* 2. The foregoing conſtruction for determining the angle CBF is Dr. *Halley*'s[c], and Sir *Iſaac Newton*'s rule for calculating it, is this that follows. As $\sqrt{\overline{mm - 1} \times RR}$ is to $\sqrt{II - RR}$, ſo is the tabular radius to the coſine of the angle of incidence CBF. Whence this angle and its ſine are given by the tables, and from thence by the ratio of I to R the tabular ſine of the angle of refraction and the angle itſelf are alſo given. The rule may thus be demonſtrated. We had $CA : CB :: I : R$ and $FA : FB :: m : 1$. Hence $CA^2 = \frac{II}{RR} CB^2$, and $AF^2 = mm BF^2$; and ſo $\frac{II}{RR} CB^2 - mm BF^2 = (CA^2 - AF^2 = FC^{2}$[d]$=) CB^2 - BF^2$. Hence $\frac{II}{RR} CB^2 - CB^2 = mm BF^2 - BF^2$, and $\overline{II - RR} \times CB^2 = \overline{mm - 1} \times RR \times BF^2$; and by reſolving this equality into a proportion, and by extracting the roots, we have $\sqrt{\overline{mm - 1} \times RR} : \sqrt{II - RR} :: CB : BF ::$ radius : confine ang. CBF.

[c] Phil. Tranſ. N°. 297.

[d] Euc. I. 47.

PRO-

PROPOSITION IV.

To explain the Phænomena of the Rain-bow.

Deſign. 253. Having premiſed ſuch mathematical principles as are neceſſary for an exact computation of the apparent diameters and breadths of the Rain-bows, I will here ſubjoin Sir *Iſaac Newton*'s entire explication of the colours of the bows, and of the manner in which they are formed; taking the liberty here and there of making a few additions to it; for the ſake of ſuch readers as may not be ſo ſkilful as thoſe that he generally writes to.

Newt. Opt. p. 147. 254. This bow never appears but where it rains in the ſun ſhine, and may be made artificially by ſpouting up water which may break aloft, and ſcatter into drops, and fall down like rain. For the ſun ſhining upon theſe drops, certainly cauſes the bow to appear to a ſpectator ſtanding in a due poſition to the rain and ſun. And hence it is now agreed upon that this bow is made by refraction of the ſun's light in drops of falling rain. This was underſtood by ſome of the ancients, and of late more fully diſcovered and explained by the famous *Antonius de Dominis* archbiſhop of *Spalato* in his book *de Radiis Viſus & Lucis* publiſhed by his friend *Bartolus* at *Venice* in the year 1611, and written above 20 years before. For he teaches there how the interior bow is made in round drops of rain by two refractions of the ſun's light, and one reflection between them; and the exterior bow by two refractions and two ſorts of reflections between them in each drop of water; and proves his explications by experiments made with a phial full of water, and with globes of glaſs filled with water, and placed in the ſun to make the colours of the two bows appear in them. The ſame explication *Des-Cartes* has purſued in his *Meteors* and mended that of the exterior bow. But while they underſtood not the true origin of colours, it is neceſſary to purſue it a little farther.

255. For underſtanding therefore how the bow is made, let a drop of rain or any other ſpherical tranſparent body be repreſented by the ſphere $BNFG$ deſcribed with the center C and ſemidiameter CN. And let AN be one of the ſun's rays incident upon it at N, and be thence refracted to F, where let it either go out of the ſphere by refraction toward V, or be reflected to G; and at G let it either go out by refraction to R, or be reflected to H; and at H let it go out by refraction towards S, cutting the incident ray in Y. Produce AN and RG till they meet in X, and upon AX and NF let fall the perpendiculars CD and CE, and produce CD till it falls (Fig. 197.)

upon

upon the circumference at L. Parallel to the incident ray AN draw the diameter BQ; and let the ſine of incidence out of air into water, be to the ſine of refraction as I to R. Now if you ſuppoſe the point of incidence N to move from the point B continually till it comes to L, the arch QF will firſt increaſe and then decreaſe, and ſo will the angle AXR which the rays AN and GR contain; and the arch QF and angle AXR will be biggeſt when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{3RR}$[a], in which caſe NE will be to ND as $2R$[a] to I[b]. Alſo the angle AYS, which the rays AN and HS contain[b] will firſt decreaſe and then increaſe; and grow leaſt when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{8RR}$, in which caſe NE will be to ND as $3R$ to I; and ſo the angle which the next emergent ray (that is the emergent ray after three reflections) contains with the incident ray AN will come to its limit, when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{15RR}$; in which caſe NE will be to ND as $4R$ to I. And the angle which the ray next after that emergent, (that is the ray emergent after four reflections) contains with the incident, will come to its limit, when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{24RR}$; in which caſe NE will be to ND as $5R$ to I; and ſo on infinitely, the numbers 3, 8, 15, 24, &c. being gathered by continual addition of the terms of the arithmetical progreſſion 3, 5, 7, 9, &c.

[a] Art. 252.
[b] Art. 246. 249.

256. Now it is to be obſerved, that as when the ſun comes to his tropicks, days increaſe and decreaſe but a very little for a great while together; ſo when by increaſing the diſtance CD, theſe angles come to their limits, they vary their quantity but very little for ſome time together; and therefore a far greater number of rays which fall upon all the points N in the quadrant BL ſhall emerge in the limits of theſe angles than in any other inclinations. Add Fig. 198. to this, that of all the rays which fall upon the quadrant BL, thoſe contiguous ones can only emerge parallel to one another, which emerge in the limits of theſe angles; and that all other contiguous rays emerge diverging from points either behind or before the drop; and conſequently will fall much thinner upon the eye, at a great diſtance from the drop, than the parallel rays. For thoſe rays which converge to points behind the eye, placed at a great diſtance from a ſmall drop, are not ſenſibly different from parallel rays. This will appear by obſerving that while the arch BN is continu- Fig. 197. ally increaſing from nothing, and the angle AXR, for example, is alſo increaſing; the ſucceſſively emergent rays, being continually leſs and leſs inclined to the incident ones or to the fixt line BQ, are alſo ſucceſſively inclined in ſmall angles to one another; and the ſame

ſame thing is manifeſt while the angle AXR is decreaſing; the ſucceſſive rays being more and more inclined to BQ; conſequently in the limit between the increaſe and decreaſe of this angle the contiguous incident rays muſt emerge parallel to one another.

257. And farther it is to be obſerved, that the rays which differ in refrangibility will have different limits of their angles of emergence; and by conſequence according to their different degrees of refrangibility, emerge moſt copiouſly in different angles; and being ſeparated from one another appear each in their proper colours. Add to this that although the heterogeneal rays of any ſlender pencil whatever, as AN, will be ſeparated by refractions at the drop

Fig. 199. into rays $NFGR$ of one colour, and $Nfgr$ of another, as by refractions through a priſm; yet theſe emergent rays GR, gr will not affect the eye with their diſtinct colours, unleſs they be in the limits of the angles AXR Axr; becauſe every where within theſe greateſt angles, an infinite number of ſuch coloured pencils being variouſly inclined to one another are mixt together, and conſequently appear white or without diſtinct colours. And the ſame may be ſaid of the rays emerging any where within the greateſt angle NYS. Fig. 197.

258. Now what theſe angles are may be gathered firſt by computing the angles of incidence and refraction by art. 252, and then the angles AXG, AYS themſelves by the 248th article. For in the

[a] Art. 31. leaſt refrangible rays the ſines I and R are 108 and 81[a], and thence by computation the greateſt angle AXR will be found 42 degrees and 2 minutes; and the leaſt angle AYS, 50 degrees and 57 minutes. And in the moſt refrangible rays the ſines I and R are 109 and 81, and thence by computation the greateſt angle AXR will be found 40 degrees and 17 minutes, and the leaſt angle AYS, 54 degrees and 7 minutes.

Fig 200. 259. Suppoſe now that O is the ſpectator's eye, and OP a line drawn parallel to the ſun's rays. Let POE, POF, POG, POH be angles of 40 degr. 17 min.; 42 deg. 2 min.; 50 deg. 57 min.; 54 deg. 7 min. reſpectively; and theſe angles turned about their common ſide OP, ſhall with their other ſides OE, OF, and OG, OH deſcribe the verges of two rain-bows $AFBE$ and $CHDG$. For if E, F, G, H be drops placed any where in the conical ſuperficies deſcribed by OE, OF, OG, OH and be illuminated by the ſun's rays SE, SF, SG, SH; the angle SEO being equal to POE or 40 deg. 17 min. ſhall be the greateſt angle in which the moſt refrangible rays can after one reflection be refracted to the eye; and therefore all the drops in the line OE ſhall ſend the moſt refrangible rays moſt copiouſly

copiously to the eye; and thereby strike the senses with the deepest violet colour in that region. In like manner the angle *SFO* being equal to the angle *POF* or 42 deg. 2 min. shall be the greatest in which the least refrangible rays after one reflection can emerge out of the drops; and therefore these rays shall come most copiously to the eye from the drops in the line *OF*, and strike the senses with the deepest red colour in that region. And by the same argument the rays which have intermediate degrees of refrangibility shall come most copiously from drops between *E* and *F* and strike the senses with the intermediate colours in the order which their degrees of refrangibility require; that is in the progress from *E* to *F* or from the inside of the bow to the outside in this order, violet, indigo, blue, green, yellow, orange, red. But the violet by the mixture of the white light of the clouds will appear faint and incline to purple. It may be farther observed, that all the rays but the violet in the line *SE* will emerge from *E* in a greater angle than *SEO* made by the violet, and consequently will pass below the eye; and that all the rays but the red in the line *SF* will emerge from *F* in a lesser angle than *SFO* made by the red, and consequently will pass above the eye; by which means only red will appear in the line *SF* and only violet in the line *SE*.

260. Again the angle *SGO* being equal to the angle *POG* or 50 deg. 57 min. shall be the least angle in which the least refrangible rays can after two reflections emerge out of the drops; and therefore the least refrangible rays shall come most copiously to the eye from the drops in the line *OG*, and strike the sense with the deepest red in that region. And the angle *SHO* being equal to the angle *POH* or 54 deg. 7 min. shall be the least angle in which the most refrangible rays after two reflections can emerge out of the drops; and therefore these rays shall come most copiously to the eye from the drops in the line *OH*, and strike the sense with the deepest violet in that region. And by the same argument the drops in the regions between *G* and *H* shall strike the sense with intermediate colours in the order which their degrees of refrangibility require; that is in the progress from *G* to *H*, or from the inside of the bow to the outside in this order; red, orange, yellow, green, blue, indigo, violet. And since these four lines *OE*, *OF*, *OG*, *OH* may be situated any where in the abovementioned conical surfaces, what is said of the drops and colours in these lines is to be understood of the drops and colours every where in those surfaces.

261. Thus shall there be made two bows of colours, an interior and stronger by one reflection in the drops, and an exterior and

fainter by two; for the light becomes fainter by every reflection. And their colours shall lye in a contrary order to one another; the red of both bows bordering upon the space *GF*, which is between the bows. The apparent breadth of the interior bow *EOF*, measured cross the colours, shall be 1 deg. 45 min. and the breadth of the exterior, *GOH*, shall be 3 deg. 10 min. and the apparent distance between them, *GOF*, shall be 8 deg. 55 min. the greatest semidiameter of the innermost, that is, the angle *POF* being 42 deg. 2 min. and the least semidiameter of the outermost, *POG*, being 50 deg. 57 min.

262. These are the measures of the bows as they would be were the sun but a point, for by the breadth of his body the breadths of the bows will be increased and their distance decreased by half a degree. And so the breadth of the interior Iris will be 2 deg. 15 min. that of the exterior 3 deg. 40 min., their distance 8 deg. 25 min., the greatest semidiameter of the interior bow 42 deg. 17 min., and the least of the exterior 50 deg. 42 min. For let *SEO* be the limit of all the angles under the rays of any one colour, which coming from the center of the sun are reflected from the drop at *E* to the eye at *O*. In the ray *SE* take any point *S* at pleasure, and let the angles *ESM*, *ESN* and also *EOM*, *EON* be severally equal to a quarter of a degree, that is to half the apparent breadth of the sun. And joining *OS*, since the sums of the angles at the base *OS*, of the several triangles *OSM*, *OSE*, *OSN*, are equal among themselves, their vertical angles at *M*, *E*, *N* are also equal among themselves. Consequently the angle *SMO* will be the limit of all the angles contained under the incident and emergent rays of the same colour as before, which came from *m* the highest point of the sun; and *SNO* the limit of all the angles contained under the incident and emergent rays of the same colour as before, which came from *n* the lowest point of the sun. Therefore if all the rays of the sun were of the same colour, or alike refrangible, the apparent breadth of the bow, measured by the angle *MON*, would be but half a degree or equal to the apparent breadth of the sun measured by the angle *MSN* or *mSn*. But since his rays are differently refrangible, conceive the drop *E* to be placed any where in the inward or outward verges of the bows above described, upon supposition that the sun was but a point; and then it is manifest that the angle *EOM* must be added to the inside, and *EON* to the outside of the angles which the breadths of those bows subtend at *O*, to obtain their apparent breadths. A rain-bow is therefore a circular image of the sun reflected to the eye from the farther surfaces of innumerable drops of

Fig. 201.

of falling rain, and dilated in breadth by the unequal refrangibility of rays of different colours.

263. And such are the dimenſions of the bows in the heavens found to be very nearly, when their colours appear ſtrong and perfect. For once by ſuch means as I then had I meaſured the greateſt ſemidiameter of the interior iris about 42 degrees, the breadth of the red, yellow, green in that iris 63 or 64 minutes, beſides the outmoſt faint red obſcured by the brightneſs of the clouds, for which we may allow 3 or 4 minutes more. The breadth of the blue was about 40 minutes more beſides the violet, which was ſo much obſcured by the brightneſs of the clouds that I could not meaſure its breadth. But ſuppoſing the breadth of the blue and violet together to equal that of the red, yellow and green together; the whole breadth of this iris will be about $2\frac{1}{4}$ degrees, as above. The leaſt diſtance between this iris and the exterior iris was about 8 degrees and 30 minutes. The exterior iris was broader than the interior, but ſo faint, eſpecially on the blue ſide, that I could not meaſure its breadth diſtinctly. At another time when both bows appeared more diſtinct I meaſured the breadth of the interior iris 2 deg. 10 min. and the breadth of the yellow and green in the exterior iris was to the breadth of the ſame colours in the interior as 3 to 2.

264. Whoever has a mind to repeat theſe obſervations after Sir *Iſaac Newton* may obſerve, that the apparent ſemidiameter of the bow, (or of any ring of colours in either of the bows) is equal to the apparent altitude of its higheſt point added to the ſun's altitude, and conſequently may be meaſured by a common quadrant. For let SOP be the axis of the bows paſſing through the ſun at S Fig. 202. and the eye at O, GOH an horizontal line, E the higheſt point of any ring of either of the bows, whoſe apparent ſemidiameter EOP is required. It is manifeſt that the angle $EOP = EOH + HOP = EOH + SOG$.

265. This explication of the rain-bow is yet farther confirmed by the known experiment (made by *Antonius de Dominis* and *Des Cartes*) of hanging up any where in the ſun-ſhine a glaſs-globe filled with water, and viewing it in ſuch a poſture that the rays which come from the globe to the eye may contain with the ſun's rays an angle of either 42 or 50 degrees. For if the angle be about 42 or Fig. 200. 43 degrees the ſpectator ſuppoſed at O, ſhall ſee a full red colour in that ſide of the globe which is oppoſed to the ſun; as it is repreſented at F: and if that angle be made leſs, ſuppoſe by depreſſing the globe to E, there will appear other colours yellow, green and blue ſucceſſively in the ſame ſide of the globe. But if the angle be

made about 50 degrees, ſuppoſe by lifting up the globe at G, there will appear a red colour in that ſide of the globe which lyes toward the ſun: and if the angle be made greater, ſuppoſe by lifting up the globe to H, the red will turn ſucceſſively to other colours, yellow, green and blue. The ſame thing I have tried by letting a globe reſt, and by raiſing or depreſſing the eye, or otherwiſe moving it to make the angle of a juſt magnitude. So far Sir *Iſaac Newton*.

LEMMA III.

266. *The tangent of the ſum of two angles, is to the ſum of their tangents, as the ſquare of the radius, to the ſquare of the radius diminiſhed by the rectangle under the tangents: and the tangent of the difference of two angles, is to the difference of their tangents, as the ſquare of the radius, to the ſquare of the radius increaſed by the rectangle under the tangents.*

Fig. 203, 204. Let RA and RB be tangents of two angles ROA, ROB. Then as AB, the ſum or difference of the tangents, is to AO, the ſecant of either of the angles, ſo let AO be to AC, to be placed from A towards B. Again as RC is to RO, ſo let RO be to RD; and RD will be the tangent of the ſum or difference of the two angles ROA, ROB. For joining CO, by the firſt of theſe proportions the triangles AOB, ACO will be equiangular [a]; and ſo the angle AOB is equal to ACO, or to ROD by the ſecond proportion [b]. Hence in fig. 203, becauſe $AC = \frac{AOq}{AB} = \frac{RAq + ROq}{RB + RA}$, we have $RC = (AC - AR =) \frac{RAq + ROq}{RB + RA} - RA = \frac{ROq - RB \times RA}{RB + RA}$; whence $RD = \frac{RB + RA}{ROq - RB \times RA} \times ROq$. By a like proceſs fig. 204, we have $AC = \frac{RAq + ROq}{RB - RA}$; whence $RD = \frac{RB - RA}{ROq + RB \times RA} \times ROq$. *Q. E. D.*

[a] Euc. VI. 6.
[b] Euc. VI. 8.

267. *Corol.* 1. Hence the tangent of the ſum of any number of given angles, or the tangent of any multiple of a given angle, may be eaſily computed. Put $RO = r$, $RA = a$, $RB = b$, then the tangent of the ſum of the angles whoſe tangents are a and b, that is $RD = \frac{b + a}{rr - ab} \times rr$; call this tangent x; then for the ſame reaſon, the tangent of the ſum of this laſt angle and of a third angle, whoſe

whose tangent is c, is $\frac{x+c}{rr-xc} \times rr$ or (by substituting the value of x) $\frac{rr \times \overline{a+b+c} - abc}{rr - ab - ac - bc}$, the tangent of the sum of three angles whose tangents are a, b, c; and so on.

268. *Corol.* 2. Now put $a = b = c$; and for the tangent of a double angle we have $\frac{2a}{rr-aa}rr$; and for the tangent of a treble angle $\frac{3arr - a^3}{rr - 3aa}$; and so on.

PROPOSITION V.

269. *The apparent semidiameter of any rain-bow, or the greatest angle under an incident and emergent ray after any given number of successive reflections, being given; to find the ratio of refraction.*

Let m be the given number of successive reflections increased by an unite, and supposing the angles ABC, ABD to be the angles of incidence and refraction sought, let the angle $ABE = m \times ABD$, and the angle CBE, or $m \times ABD - ABC$, will be half the given angle under the incident and the emergent ray after $m-1$ reflections[a]. Put the common radius $AB = r$, the unknown tangent of refraction $AD = a$, and the tangent of incidence $AC = ma$[b], also $AE = x$, and t for the tangent of the given angle CBE answering to the radius r. Then by the lemma $t : x - ma :: rr : rr + xma$; whence $t = \frac{x - ma}{rr + xma} rr$. Fig. 205

[a] Art. 248.

[b] Art. 251.

Case 1. In the first rain-bow $m = 2$; whence $t = \frac{x - 2a}{rr + 2xa} rr$, and by art. 268, $x = \frac{2a}{rr - aa} rr$ the tangent of $2ABD$. Substitute this value for x in the former equation and by reduction it becomes $a^3 - \frac{3}{2}taa - \frac{1}{2}trr = 0$. By resolving this equation the tangent a of the angle of refraction will be given, and the tangent of the angle of incidence $AC = 2a$ by art. 251, whence the ratio of their sines is given by the tables.

Case 2. In the second rain-bow $m = 3$, whence $t = \frac{x - 3a}{rr + 3xa} rr$, and by art. 268, $x = \frac{3arr - a^3}{rr - 3aa}$, the tangent of $3ABD$. Substitute

made about 50 degrees, ſuppoſe by lifting up the globe at G, there will appear a red colour in that ſide of the globe which lyes toward the ſun: and if the angle be made greater, ſuppoſe by lifting up the globe to H, the red will turn ſucceſſively to other colours, yellow, green and blue. The ſame thing I have tried by letting a globe reſt, and by raiſing or depreſſing the eye, or otherwiſe moving it to make the angle of a juſt magnitude. So far Sir *Iſaac Newton*.

LEMMA III.

266. *The tangent of the ſum of two angles, is to the ſum of their tangents, as the ſquare of the radius, to the ſquare of the radius diminiſhed by the rectangle under the tangents: and the tangent of the difference of two angles, is to the difference of their tangents, as the ſquare of the radius, to the ſquare of the radius increaſed by the rectangle under the tangents.*

Fig. 203, 204. Let RA and RB be tangents of two angles ROA, ROB. Then as AB, the ſum or difference of the tangents, is to AO, the ſecant of either of the angles, ſo let AO be to AC, to be placed from A towards B. Again as RC is to RO, ſo let RO be to RD; and RD will be the tangent of the ſum or difference of the two angles ROA, ROB. For joining CO, by the firſt of theſe proportions the triangles AOB, ACO will be equiangular[a]; and ſo the angle AOB is equal to ACO, or to ROD by the ſecond proportion[b]. Hence in fig. 203, becauſe $AC = \frac{AOq}{AB} = \frac{RAq + ROq}{RB + RA}$, we have $RC = (AC - AR =) \frac{RAq + ROq}{RB + RA} - RA = \frac{ROq - RB \times RA}{RB + RA}$; whence $RD = \frac{RB + RA}{ROq - RB \times RA} \times ROq$. By a like proceſs fig. 204, we have $AC = \frac{RAq + ROq}{RB - RA}$; whence $RD = \frac{RB - RA}{ROq + RB \times RA} \times ROq$. *Q. E. D.*

[a] Euc. VI. 6.
[b] Euc. VI. 8.

267. *Corol.* 1. Hence the tangent of the ſum of any number of given angles, or the tangent of any multiple of a given angle, may be eaſily computed. Put $RO = r$, $RA = a$, $RB = b$, then the tangent of the ſum of the angles whoſe tangents are a and b, that is $RD = \frac{b + a}{rr - ab} \times rr$; call this tangent x; then for the ſame reaſon, the tangent of the ſum of this laſt angle and of a third angle, whoſe

whose tangent is c, is $\frac{x+c}{rr-xc} \times rr$ or (by substituting the value of x) $\frac{rr \times \overline{a+b+c} - abc}{rr-ab-ac-bc}$, the tangent of the sum of three angles whose tangents are a, b, c; and so on.

268. *Corol.* 2. Now put $a = b = c$; and for the tangent of a double angle we have $\frac{2a}{rr-aa} rr$; and for the tangent of a treble angle $\frac{3arr - a^3}{rr - 3aa}$; and so on.

PROPOSITION V.

269. *The apparent semidiameter of any rain-bow, or the greatest angle under an incident and emergent ray after any given number of successive reflections, being given; to find the ratio of refraction.*

Let m be the given number of successive reflections increased by an unite, and supposing the angles ABC, ABD to be the angles of incidence and refraction sought, let the angle $ABE = m \times ABD$, and the angle CBE, or $m \times ABD - ABC$, will be half the given angle under the incident and the emergent ray after $m - 1$ reflections[a]. Put the common radius $AB = r$, the unknown tangent of refraction $AD = a$, and the tangent of incidence $AC = ma$[b], also $AE = x$, and t for the tangent of the given angle CBE answering to the radius r. Then by the lemma $t : x - ma :: rr : rr + xma$; whence $t = \frac{x - ma}{rr + xma} rr$. Fig. 205

[a] Art. 248.

[b] Art. 251.

Case 1. In the first rain-bow $m = 2$; whence $t = \frac{x-2a}{rr+2xa} rr$, and by art. 268, $x = \frac{2a}{rr-aa} rr$ the tangent of $2ABD$. Substitute this value for x in the former equation and by reduction it becomes $a^3 - \frac{3}{2} taa - \frac{1}{2} trr = 0$. By resolving this equation the tangent a of the angle of refraction will be given, and the tangent of the angle of incidence $AC = 2a$ by art. 251, whence the ratio of their sines is given by the tables.

Case 2. In the second rain-bow $m = 3$, whence $t = \frac{x-3a}{rr+3xa} rr$, and by art. 268, $x = \frac{3arr - a^3}{rr - 3aa}$, the tangent of $3ABD$. Sub-

stitute

ſtitute this value for x and you will find $a^4 + \frac{8}{3}\frac{rr}{t}a^3 - 2rraa * - \frac{1}{3}r^4 = o$; or putting $T = \frac{rr}{t}$ the tangent of half the angle of this bow[a], $a^4 + \frac{8}{3}Ta^3 - 2rraa * - \frac{1}{3}r^4 = o$. The ſame method ſerves for other bows to infinity.

[a] Art. 248.

270. *Corol.* In the firſt caſe putting T for $2a$ or AC the tangent of the angle of incidence, and ſubſtituting $\frac{1}{2}T$ for a in the former equation $a^3 - \frac{3}{2}taa - \frac{1}{2}trr = o$, it is changed to this $T^3 - 3tTT - 4rrt = o$, the ſame as Dr. *Halley*'s, who propoſed this problem as an expeditious method for finding the ratio of refraction in any fluid, by obſerving (when the ſun is low and ſhines very bright) the angle under an incident and the emergent ray from a drop of any fluid hanging at the end of a capillary tube. See his examples Phil. Tranſ. N°. 267, and alſo the Rev^d^. Dr. *Morgan*'s Diſſertation upon the Rain-bow among the Notes upon *Rohault*'s Phyſicks. P. 3. Ch. 17.

CHAP. XIV.

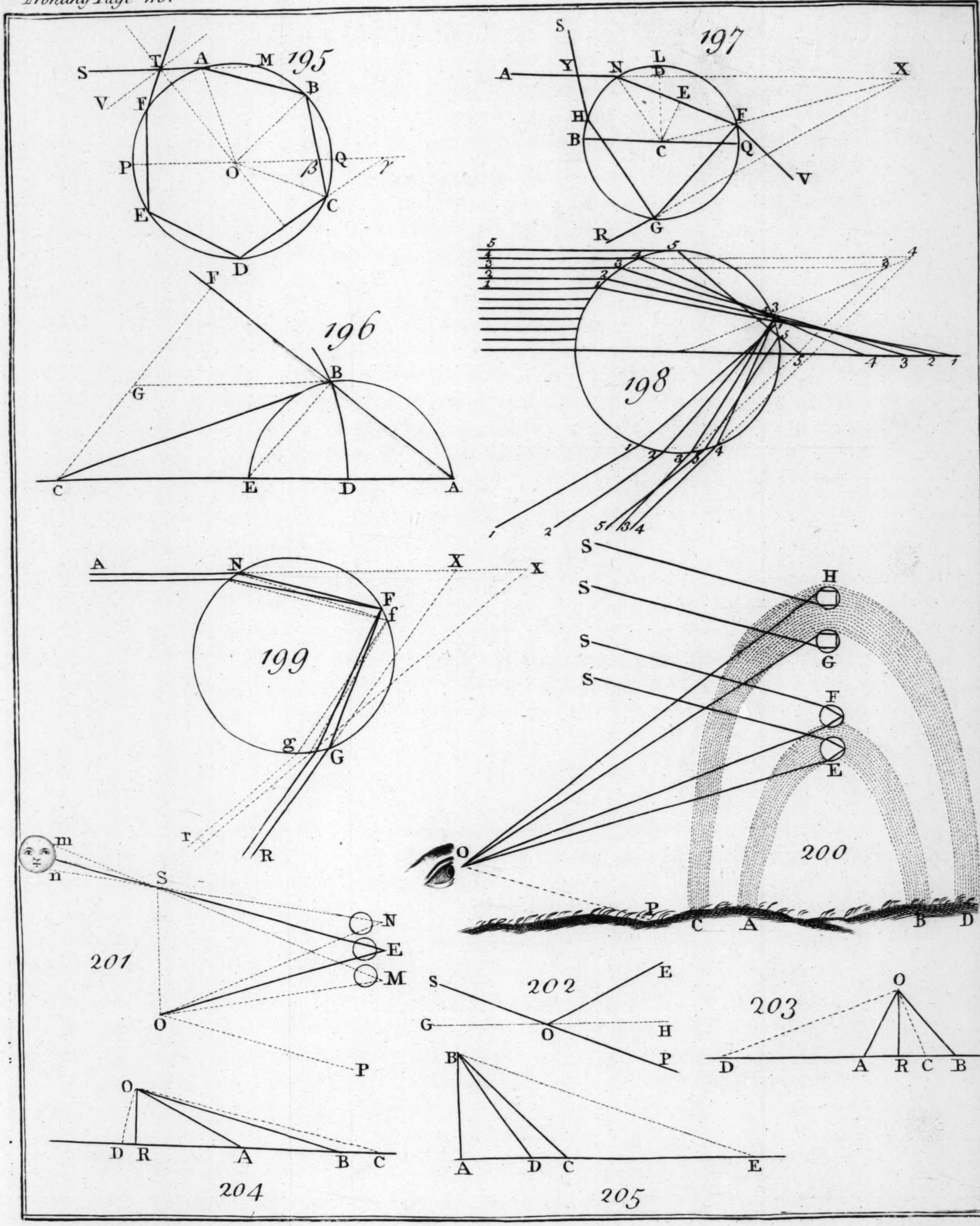
TELESCOPICAL DISCOVERIES IN THE FIXT STARS.

Multitudes of Teleſcopick Stars.

271. THAT the fixt ſtars have no ſenſible parallax, or, which is the ſame thing, that the earth's annual orbit (whoſe diameter a cannon ball could not deſcribe in leſs than 50 years,) would appear of no ſenſible magnitude through a teleſcope placed at a fixt ſtar, is ſuch an amazing concluſion as could not be believed, were it not ſupported by undeniable evidence. But as this is the caſe, it is no longer a wonder why the beſt teleſcopes do not at all magnify the apparent diameters of the fixt ſtars, though they diſcover vaſt multitudes that are quite imperceptible to the naked eye; and the more of them as the aperture is more enlarged to take in more light, and the eye-glaſs made flatter to render it diſtinct[a]. The Milky Way, which had puzzled the ancient philoſophers for many ages, was found at laſt to be nothing elſe but a prodigious number of very minute ſtars, ſo cloſe to one another that the naked eye can only perceive a whitiſh mixture of their faint lights. This was *Galileo*'s diſcovery, who found alſo that thoſe faint ſtars, which Aſtronomers call *Nebuloſæ*, appeared through his teleſcope to be ſmall cluſters of very minute ſtars.

[a] Art. 240.

272. *Hu-*



272. *Hugenius* in the year 1656, looking by chance through a large telescope, at three small stars very close to one another in the middle of Orion's sword, saw several more as usual. But the three little stars very near one another (marked θ by *Bayer*), together with four more, shone out as it were through a whitish cloud, much brighter than the ambient sky: which being very black and serene caused that lucid part to appear like an aperture, that gave a prospect into a brighter region. He viewed it many times, and found it continued in the very same place, and of the same shape as the figure represents, and called it *Portentum cui certe simile aliud nusquam apud reliquas fixas potuit animadvertere* [a].

Lucid spots among the fixt Stars.

Fig. 206.

273. But in the Philosophical Transactions [b], there is an account of a later discovery of five more such lucid spots, though less considerable than this of *Hugenius*; the middle of which, we are there told, is at present in ♊. 19°. 00′. with south latitude 28°. 45′; and that it sends forth a radiant beam into the south east, as another in the girdle of *Andromeda* seems to do into the north east. It is also there remarked, that "though these spots are in appearance but small, and most of them but a few minutes in diameter, yet since they are among the fixt stars, as having no annual parallax, they cannot fail to occupy spaces immensely great, and perhaps not less than our whole solar system; in all which spaces it should seem, that there is a perpetual uninterrupted day."

274. It is to the Author of these reflections, if I mistake not, that we owe another curious account of what is principally remarkable in the new stars that have appeared and disappeared for 150 years last past [c]. I will mention but one or two. "That in the chair of *Cassiopeia* was not seen by *Cornelius Gemma* on the eighth of November 1572, who says, he that night considered that part of the Heaven in a very serene sky, and saw it not: but that the next night, Novemb. 9, it appeared with a splendour exceeding all the fixt stars and scarce less bright than Venus. This was not seen by *Tycho Brahe* before the 11th of the same month; but from thence he assures us, that it gradually decreased and died away, so as in March 1574, after 16 months, to be no longer visible; and at this day not the least signs of it remain. The place thereof in the sphere of the fixt stars, by the accurate observations of the same *Tycho*, was 0s. 9°. 17′. from the first star of Aries, with 53°. 45′. north latitude." To this account Sir *Isaac Newton* adds [d], that in November, when it first appeared, it seemed equal to Venus in brightness, in December to Jupiter, in January 1573 less than Jupiter, but bigger than Sirius, and equal to him in February and March; in April and May equal to the

New Stars.

[a] Systema Saturnium p. 8.

[b] N. 347. Jones's abr. Vol. 4. p. 224.

[c] Phil. Trans. N. 346. Jones's abr. Vol. 1. p. 222.

[d] Philos. Princip. p. 526.

the ſtars of the ſecond magnitude, in June, July and Auguſt to thoſe of the third, in September, October and November to thoſe of the fourth, in December and January 1574 to thoſe of the fifth, in February to thoſe of the ſixth, and in March it vaniſhed. That its colour was at firſt clear, white and ſplendid, afterwards yellow, and in March 1573 red and fiery like Mars or Aldebaran, in May of a pale livid colour like that of Saturn, which grew fainter and fainter till it vaniſhed.

275. " That ſuch another ſtar was ſeen and obſerved by the ſcholars of *Kepler* to begin to appear on Sept. 30. *St. Vet. Anno* 1604, which was not to be ſeen the day before; but it broke out at once with a luſtre greater than that of Jupiter; and like the former, died away gradually, and in much about the ſame time diſappeared totally, there remaining no footſteps thereof in January $160\frac{5}{6}$. This ſtar was near the ecliptick, following the right leg of *Serpentarius*; and by the obſervations of *Kepler* and others, was in $7^s. 20^o. 00'$. from the firſt ſtar of *Aries*, with north latitude $1^o. 56'$.

276. Laſtly, that the ſudden eruption of ſuch another ſtar, ſhining out more than uſual, engaged *Hipparchus* to make the firſt catalogue of the fixt ſtars; that poſterity might know what changes might happen among them.

The Origine of new Stars.

277. How minute ſoever the particles of light may be, the *perpetual* emiſſion of them from the body of the ſun muſt have cauſed, before this time, a ſenſible diminution in his magnitude, without ſome ſupply of new matter. But ſince a diminution of the ſun's diameter has not yet been diſcovered by the moſt accurate obſervations, Sir *Iſaac Newton* therefore imagined, that thoſe comets which approach ſo near the ſun as to paſs through his atmoſphere, may be ſo much reſiſted and retarded after ſeveral revolutions, as at laſt to fall upon the ſun and ſo become a mean of keeping his magnitude nearly the ſame. And this opinion led him farther to conjecture, that the ſtars we have mentioned, which ſuddenly ſhine out with very great ſplendor and then decay gradually till they vaniſh out of ſight, may now and then be ſtirred up and blaze out again by the ſhock of a comet falling down upon them. But thoſe other new ſtars, which appear and diſappear periodically, which increaſe by very ſlow degrees, and ſeldom exceed the ſtars of the third magnitude (ſeveral of which may be ſeen in the hiſtory I mentioned) he takes to be of another ſort, or at leaſt in another ſtate; which revolving about their axes, like the ſun, may expoſe their light and dark parts to us ſucceſſively. For the fixt ſtars are undoubtedly ſelf-ſhining bodies of the ſame kind as the ſun, and therefore equally ſubject to large

dark

dark ſpots or cruſts upon their ſurfaces. Becauſe the light of the ſun propagated to thoſe vaſt diſtances, and reflected back from opake bodies of no ſenſible apparent magnitudes, would be too much rarified to affect our ſenſes; as *Galileo* collected from the fainter lights of the remoter planets from the ſun, compared to the luſtre of the fixt ſtars.

An enquiry into the annual parallax of the fixt ſtars.

278. After ſeveral attempts by Dr. *Hook*, Mr. *Flamſtead*[a], and others[b], to determine the annual parallax of the fixt ſtars, the honourable *Samuel Molyneux* Eſq; in the year 1725, erected at *Kew* a very accurate inſtrument, in order if poſſible to arrive at ſome degree of certainty in this difficult enquiry: in the proſecution of which he followed Dr. *Hook* in ſome reſpects, as in taking the zenith diſtances of the brighteſt ſtar in the Dragon's head at the times of its tranſits over the meridian, and alſo in the form of his inſtrument, conſtructed almoſt upon the ſame principles with the Doctor's, but executed to a degree of exactneſs vaſtly greater, and chiefly owing to the care and contrivance of Mr. *George Graham*.

[a] Walliſii opera, Vol. 3. p. 701.
[b] Phil. Tranſ. N. 364. Abridg. Vol. 6. p. 165.

279. The Rev. Mr. *Bradley*, Profeſſor of Aſtronomy at Oxford, who all along aſſiſted Mr. *Molyneux* in the proſecution of this noble deſign, has obliged the publick with a very accurate hiſtory of it, in a letter to D. *Halley*[c]; containing not only an account of ſeveral new and ſurpriſing phænomena that attended the obſervations, (which he therefore continued and repeated after Mr. *Molyneux*'s deceaſe,) but alſo a compleat diſcovery of the true cauſe of them; which at laſt enabled him to ſettle the point in queſtion, and to draw from it ſome admirable conſequences relating to the propagation of light. As I look upon theſe diſcoveries to be ſome of the fineſt that we have had ſince the invention of teleſcopes, I will endeavour to give the ſubſtance of them in as clear a manner as I can.

[c] Phil. Tranſ. N°. 406. Abridg. Vol. 6. p. 167.

Some account of the obſervations and phænomena.

280. The reſult of the obſervations upon the bright ſtar in the dragon's head, marked γ by *Bayer*, was this;

Beginning from December 3, 1725, its diſtance from the zenith being taken ſeveral days, at the time of its *tranſit* over the meridian, there appeared no material difference in the obſervations.

281. On Decem. 17, it paſſed a little more ſoutherly from the zenith than before, and ſtill more on the 20th; which was matter of ſurprize, both becauſe no ſenſible alteration of parallax could ſo ſoon be expected in this ſtar at that time of the year, and becauſe it was the contrary way to what it would have been, had it proceeded from an annual parallax.

282. About the beginning of March 1726, the ſtar was found to be 20″ more ſoutherly than at the time of the firſt obſervation, and ſeemed to have arrived at its utmoſt limit ſouthwards.

283. By the middle of April it appeared to be returning back again towards the north, and about the beginning of June it paſſed at the ſame diſtance from the zenith as it had done in December when it was firſt obſerved.

284. From that time it continued to move northwards till September following, when it again became ſtationary, being then near 20″ more northerly than in June, and no leſs than 39″ more northerly than it was in March.

285. From September it returned towards the ſouth, till it arrived in December at the ſame ſituation it was in at that time twelve months, allowing for the difference of declination on account of the preceſſion of the equinox.

286. By the like obſervations made upon a ſmall ſtar almoſt oppoſite in right aſcenſion to γ *Draconis*, and at about the ſame diſtance from the north pole of the equator, it appeared to change its declination 19″, that is about half as much as γ *Draconis* did in the ſame time. Which plainly proved, as Mr. *Bradley* obſerves, that theſe apparent changes were not owing to a nutation of the earth's axis, ſince the changes on this account would have been nearly equal in theſe ſtars, as lying near the ſolſticial colure.

287. Upon comparing the obſervations with each other it was diſcovered in both theſe ſtars, that the apparent difference of declination, reckoned from the limits above mentioned, was always nearly proportionable to the verſed ſine of the ſun's diſtance from the equinoctial points.

288. And that the whole difference of declination in theſe ſtars, was as the ſine of the latitude of each reſpectively.

Mr. *Bradley*'s hypotheſis to ſolve theſe phænomena.

289. After a year's obſervations upon many other ſtars, in different parts of the heavens, made with a new inſtrument ſet up at *Wanſted* in 1727, Mr. *Bradley* found out ſome other properties of their apparent motions; and after examining and rejecting two or three hypotheſes, by which he attempted to ſolve them, at laſt he conjectured that all theſe phænomena proceeded from the progreſſive motion of light and the earth's annual motion in her orbit. For he perceived, that if light was propagated in time, the apparent place of a fixt object would not be the ſame when the eye is at reſt, as when it is moving in any other direction than that of a line paſſing through the eye and object; and that when the eye is moving

ing

ing in different directions, the apparent place of the object would be different. I will first deduce some consequences from this hypothesis and then compare them with the phænomena.

290. If an eye moves uniformly in a straight line from *a* to *b* in the time that the light of a fixt star descends uniformly in a straight line from *c* to *b*, the star will appear in a direction constantly parallel to *ac*. Some consequences from the hypothesis. Fig. 207.

For conceiving the eye to carry the line *ac* parallel to itself, its intersection with the fixt line *bc* will move uniformly[a] from *c* to *b*, and will therefore accompany a particle of light descending uniformly from *c* to *b*; and because this intersection is a moving point, not only in the fixt line *bc*, but also in the moving line *ac*, it is plain that the particle which accompanies the intersection *c*, moves relatively in the moving line *ac*. In like manner a particle of every other ray, parallel to *cb*, which the moving line *ac* successively meets with, moves also in the moving line *ac*; and thus a succession of these particles, moving along *ac*, constitute a visual ray in whose direction the star appears.

[a] Euc. VI. 2.

291. Hence supposing the earth's center *b* to move uniformly in a circular orbit ♈ ♉ *Ab*, round the sun in its center *B*; if a line *BC* drawn towards a fixt star, supposed infinitely distant, you take a distance *BC* in proportion to *Bb* or *BA*, as the velocity of light to the velocity of the earth's center, an observer upon the earth at *b*, will constantly see that star in a direction very nearly parallel to a line *AC*, connecting the point *C* with a point *A* in the orbit constantly 90 degrees behind the earth. Fig. 208.

For drawing *ba* and *bc* parallel to *BA* and *BC* respectively, and tending the same ways from *b* and *B*, and also any line *ac* parallel to *AC*; by the similar triangles *bca*, *BCA*, we have *bc* : *ba* :: *BC* : *BA*, as the velocity of light to the velocity of the earth. Consequently if an eye be supposed to move along the tangent *ab* with this latter velocity, it will see the star in a direction constantly parallel to *ac*[b] or *AC*. But the eye in the orbit moves with that velocity, and passes by the point *b* in the direction of that tangent, and therefore at that passage it saw the star in the same direction in which the other eye in the tangent sees it constantly. The earth's diurnal motion alters this conclusion so little that I need not here consider it.

[b] Art. 290.

292. Therefore the apparent parallax of the star to an observer at *b*, is constantly measured by the angle *ACB*, if the point *A* be always 90 degrees behind the earth at *b*, and consequently 90 degrees before the sun's apparent place ☉ in the ecliptick.

 293. Hence

293. Hence the apparent latitude of any ſtar, ſuppoſed infinitely diſtant, will be leaſt of all when the ſun's place in the ecliptick is 90 degrees forwarder than the ſtar's; and from that time it will increaſe for half a year, and then deſcreaſe for the next half year; and its increment reckoned from theſe limits will be conſtantly as the verſed ſine of the ſun's longitude reckoned from his place before mentioned.

Fig. 209. For drawing CD perpendicular to the plane of the orbit, join AD, and draw DB cutting the orbit in L and M. The point L, neareſt to D, is the ſtar's place in the ecliptick, and the point ⊙, oppoſite to b, is the ſun's place therein. Now when the point ⊙ was at N, 90 degrees forwarder than L, the point A, being always 90 degrees forwarder than ⊙ [a], was at M, the fartheſt point from the perpendicular CD; and conſequently the ſtar's apparent latitude, always meaſured by the angle CAD, was then the leaſt poſſible.

[a] Art. 292.

Draw AE perpendicular to LM, and joining CL, CE, CM, draw MF perpendicular to CE produced. Then conceiving the point A to move in the perpendicular AE towards E, the angle CAD will approach to a *maximum* CED, and therefore will increaſe very little, eſpecially as the angular approach ACE is exceeding ſmall. Therefore inſtead of the apparent latitude CAD we may take CED, and conſequently the angle ECM for the increment of the leaſt latitude CMD: now this ſmall angle ECM is as its ſine MF or (becauſe the ratio of MF to ME varies very little) as ME the verſed ſine of the arch MA equal to N⊙, the ſun's longitude from N, 90 degrees forwarder than the ſtar's place L.

294. When a ſtar is ſituated any where in the ſolſticial colure, the increments or decrements of its latitude and declination are the very ſame quantities; and therefore if the ſtar be ſuppoſed infinitely diſtant, and its longitude be in the beginning of Capricorn, with north declination, its apparent declination will be the leaſt at the time of the vernal equinox, and the greateſt at the autumnal; and its increments and decrements reckoned from theſe limits, will be proportionable to the verſed ſine of the ſun's longitude reckoned from the equinoctial points: which agrees with the phænomena in Art. 287.

Fig. 210. 295. The whole apparent parallax LPM of a ſtar in the pole of the ecliptick, is to the whole apparent parallax LCM in the latitude of any other ſtar, as the radius to the ſine of the latitude CBL. For ſince BP equals BC, drawing BF and BG perpendiculars to CL and CM, the ſmall angle BPL is to BCL as BL to BF, that is, as the radius to the ſine of the angle BLF or of the latitude CBL [b]. Again, the ſmall angle BPM is to BCM as BM to BG, that is, as the

[b] Art. 68.

the radius to the ſine of *BMG* or of the latitude *CBL*, as before. Therefore the whole angle *LPM* is to *LCM* as the radius to the ſine of the latitude *CBL*.

Hence, from the obſerved parallax in latitude, or in declination of ſuch ſtars as lye in or near the ſolſticial colure, we have the parallax that would belong to a ſtar in the pole of the ecliptick, which is plainly the greateſt of all. Thus in γ *Draconis* whoſe latitude *CBL* = 74°. 58′. 20″, the obſerved parallax *LCM* was 39″[a], and thence the greateſt parallax *LPM* comes out 40″,4. Likewiſe in the little ſtar above mentioned, whoſe diſtance from the north pole of the equator is 38°. 28′. 35″[b], and conſequently its latitude a little above 28°. 02′. 25″, as being almoſt oppoſite in right aſcenſion to γ *Draconis*, the obſerved parallactick angle *LCM* was 19″[c], and thence *LPM* comes out 40″,4 as before.

[a] Art. 284.

[b] The 36th Star *Camelopardi Hevelii* in *Flamſtead*'s Catalogue.

[c] Art. 286.

The greateſt apparent parallax.

296. Mr. *Bradley* having applied his obſervations upon the parallax in declination of ſtars in any ſituation whatever, to his theory farther purſued, aſſures us they all conſpire to prove, that the greateſt parallax *LPM* is about 40 or 41 ſeconds, and thinks the *medium* 40″.$\frac{1}{2}$ cannot differ ſo much as one ſecond from the truth.

The velocity of ſtar-light.

297. Hence the velocity of ſtar-light comes out 10210 greater than the velocity of the earth's mean motion round the ſun. For the former velocity is to the latter as *BP* to *BL* or *BM*[d], that is, as the radius to the tangent of *BPL* or *BPM* = 20″$\frac{1}{4}$ as above determined.

[d] Art. 290.

Some properties of light.

298. From what has been ſaid Mr. *Bradley* infers, 1. That the lights of all thoſe ſtars arrive at the earth with equal velocities. 2. That unleſs their diſtances from us are all equal, (which for other reaſons beſides that of their different luſtre, is highly improbable) their lights are propagated uniformly to all diſtances from them. 3. That the velocity of ſtar-light is ſuch as carries it through a ſpace equal to the ſun's diſtance from us in 8′. 13″, (this time being to the time in which the earth might deſcribe that diſtance, with the velocity of her mean motion round the ſun, as 1 to 10210, and this latter time, to half a year, as the diameter of a circle to its circumference.) 4. That the time ſo determined can ſcarce differ 5 or 10 ſeconds from the truth, which is ſuch a degree of exactneſs as can never be expected from the eclipſes of Jupiter's ſatellites. 5. That as this determination of the velocity of ſtar-light, comes out a *medium* among ſeveral determinations of the velocity of the ſun's light reflected from thoſe ſatellites, we may reaſonably conclude that the velocities of theſe lights are equal. And laſtly, ſince it is highly probable that the velocity of the ſun's emitted light is alſo equal to that of ſtar-

[a] Art. 277. light[a], it is equally probable that its velocity is not altered by reflection into the same medium.

299. From Art. 291, 295, &c. it follows plainly, that a star placed in the pole of the ecliptick would appear in a year's time, to describe about the pole a little circle whose apparent semidiameter is $20''\frac{1}{4}$; and that any other star will appear to describe, about its true place, an ellipsis whose long axis is at right angles to the circle of longitude passing through the star's true place, and equal to the diameter of the little circle just mentioned, and whose short axis is to the long one, as the sine of the star's latitude to the radius *.

Real parallax of the stars insensible.

300. Upon this theory farther pursued Mr. *Bradley* proceeds synthetically, by assuming the *maximum* of apparent parallax as determined above, and calculating tables of the differences in declination of *γ Draconis* situated near the solsticial colure, and of *γ Ursæ Majoris* nearer to the equinoctial than the solsticial colure; and by com-

* The following elegant proposition is reprinted from T. Simson's Mathematical Essays, as it affords an easy solution of the consequences mentioned in this article, and serves to confirm other parts of this Theory.

PROPOSITION. *To find the path which the progressive motion of light and the motion of the Earth in its orbit make a Star appear to describe in one entire annual revolution of the Earth.*

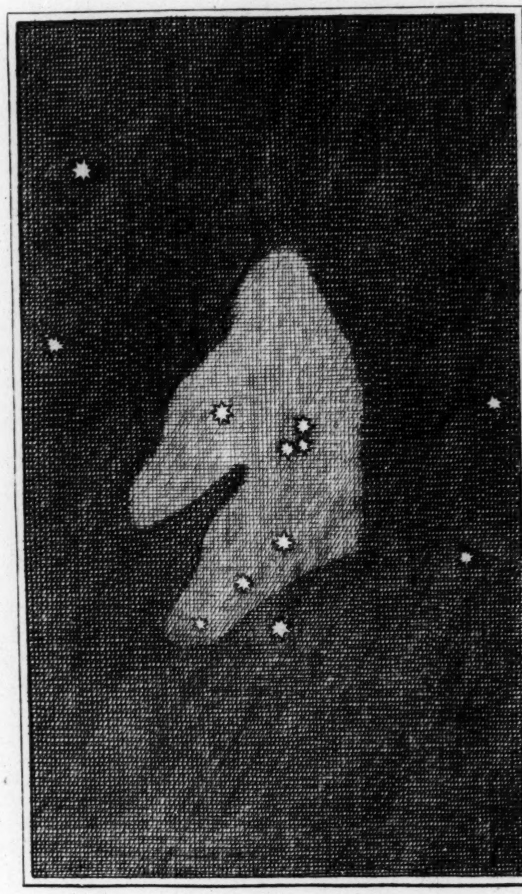
Fig. 211.

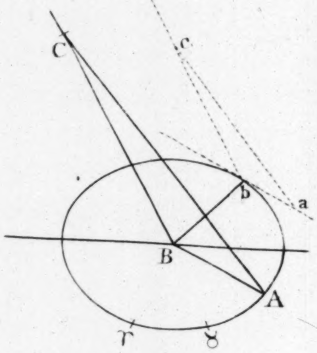
Let $ATBA$ be the orbit of the earth; S the sun in one focus; F the other focus; T the earth moving in its orbit from A towards B; DTn a tangent at T; and SD, FE perpendiculars thereto: Let $QmKRQ$ be part of an indefinite plane parallel to that of the ecliptick, passing through R the center of the given star; and take Tn to TR, as the velocity of the earth in its orbit at T to that of a particle of light coming from the said star: Let Tm be parallel to nR; PnV perpendicular to AB; and QRK parallel to PnV: Then from the 290th article it is manifest, that a ray of light, coming from R to the earth at T, will appear as if it proceeded from m, where the line Tm produced, intersects the said parallel plane, and therefore because Tm is parallel to Rn, and any parallelogram, intersecting two parallel planes, cuts them alike in every respect, it is evident that Rm must be equal to Tn, and QRm to VnD; wherefore since D and P are equal to two right angles, DSP and DnP must be equal, also, to two right angles[a], and consequently QRm ($= VnD$[b]) $= DSP = AFE$. But Tn or Rm, expressing the celerity of the earth at T, is known to be inversely as SD[c]; or because $SD \times FE$ is every where the same[d], directly as FE; wherefore the angles AFE, QRm being every where equal, and Rm in a constant proportion to FE, the curve QmK described by m, the apparent place of the star in the said parallel plane, will, it is manifest, be similar in all respects to AEB described by the point E: But this curve is known to be a circle[e]; therefore QmK must likewise be a circle, whose diameter QRK is divided by R, the true place of the star, in the same proportion as the transverse axis of the earth's orbit is divided by either of its foci. Wherefore, forasmuch as a small part of the circumjacent heavens may, in this case, be considered as a plane passing perpendicular to a line joining the eye and star, it follows from the principles of orthographic projection, that the star will be seen in the heavens as describing an ellipsis, whose center (as the excentricity of the orbit is but small) nearly coincides with the true place of the star, except the said place be in the pole or plane of the ecliptick; in the former of which cases the star will appear to describe a circle, and in the latter an arch of a great circle of the sphere, which by reason of its smallness may be considered as a right line.

[a] Eucl. I. 32. Cor. 1.
[b] Eucl XI. 10.
[c] Newt. princip. I. 1. Cor. 1.
[d] Hamilton's Conics, II. 21.
[e] Hamilton's Conics, II. 20.

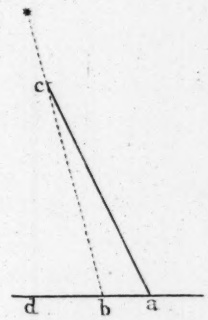
paring

206

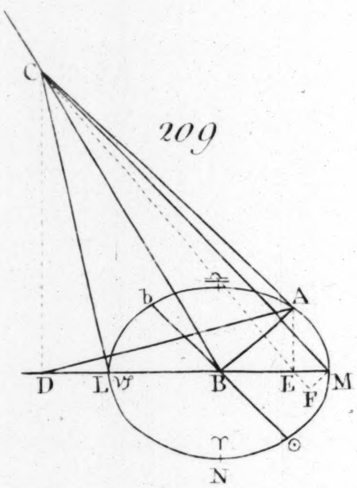




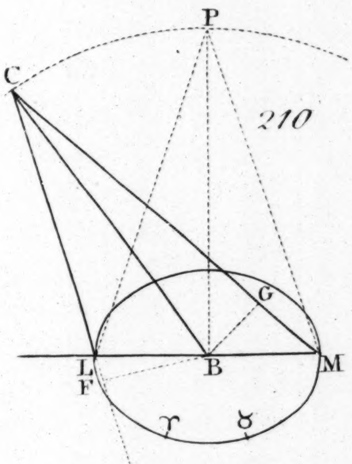
208



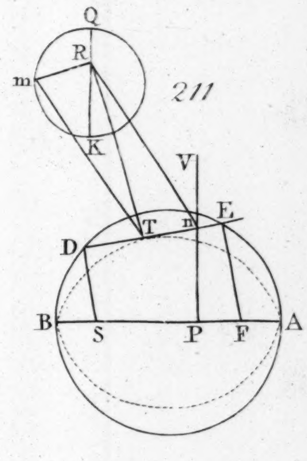
207



209



210



211

paring the tables with his obſervations, he found they agreed together throughout the year, ſometimes in the very ſame number of ſeconds, and that in 50 or 60 obſervations of each ſtar, they never differed ſo much as two ſeconds; allowing for the variation of declination cauſed by the regreſſion of the equinoctial points: which amounts to a phyſical demonſtration of the truth of his theory, and does in conſequence afford a very ſatisfactory anſwer to the point in queſtion, concerning the real parallax and diſtance of the fixt ſtars. As to which he believes he may venture to ſay, that the real parallax in either of the ſtars abovementioned does not amount to 2″, being of opinion that if it were 1″ he ſhould have perceived it in the great number of obſervations that he made eſpecially upon γ *Draconis*; which agreeing with the theory, without allowing any thing for a real parallax [a], nearly as when the ſun was in conjunction with, as in oppoſition to this ſtar, it ſeemed to him very probable, that its real parallax is not ſo great as one ſingle ſecond; and conſequently that it is above 400000 times farther from us than the ſun.

[a] Art. 290, 291, &c.

F I N I S.